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# Compact translating solitons with non-empty planar boundary

ABSTRACT

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## 1. Introduction

For a compact surface with boundary, the study of influence of the boundary on the surface is a classical topic in differential geometry. The simplest compact boundary is clearly a circle. One of long-lasting interesting problems about circular boundary is *the spherical cap conjecture*:

A constant mean curvature surface  $\Sigma \subset \mathbb{R}^3$  with a circular boundary is a planar disc or a spherical cap when  $\Sigma$  is either an immersed disc or a compact embedded surface.

There are some interesting partial results on this conjecture [5,10,12,13]. In [4], Hoffman, Rosenberg and Spruck showed that a compact constant Gaussian curvature surface spanning a circle is a spherical cap.

In this paper, we study the rigidity of translating solitons spanning a planar boundary (including a circular boundary).

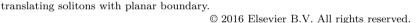
Let  $x: \Sigma^n \to \mathbb{R}^{n+1}$  be an *n*-dimensional isometric immersion, for simplicity we identify  $x(\Sigma)$  with  $\Sigma$ . For a unit vector  $v \in \mathbb{R}^{n+1}$ ,  $\Sigma$  is a *translating soliton* of the unit speed with respect to the translating direction v if  $X(p,t) = x(p) + tv : \Sigma \times \mathbb{R} \to \mathbb{R}^{n+1}$  satisfies the following mean curvature flow equation

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In this paper, we consider compact translating solitons with non-empty planar

boundary. Each boundary component lies in a plane which is orthogonal to the

translating direction. We firstly prove that when the planar boundary is either a

circle or convex and the translating soliton meets the plane containing the boundary

with a constant angle, then the compact translating soliton is part of an entire

rotationally symmetric strictly convex graphical surface. Secondly, we show that a compact translating soliton spanning two horizontal planar Jordan curves inherits

the symmetries of its boundary. We also show a balancing type formula for compact

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$$X_t(p,t) = \vec{H}(X(p,t)),$$

for all  $(p,t) \in \Sigma \times \mathbb{R}$ , that is,  $\Sigma$  satisfies  $H = \langle N, v \rangle$ , where  $\vec{H} = HN$  is the mean curvature vector.

Translating solitons arise as parabolic rescaling of Type-II singularities of the mean curvature flow [8].

The simplest nonplanar translating soliton is an entire rotationally symmetric strictly convex graphical hypersurface, U, found by Altschler and Wu [1] for n = 2, and Gui, Jian and Ju [6] for  $n \ge 3$ . It is unique up to translation.

From now on, by using an isometry, we assume that  $v = e_{n+1} = (0, ..., 1)$ . Denoting the distance function,  $r = \sqrt{x_1^2 + \cdots + x_n^2}$  for  $n \ge 2$ , U is the graph of the following ordinary differential equation with respect to  $r, x_{n+1} = u(r)$ ,

$$\frac{u''(r)}{1+u'(r)^2} + (n-1)\frac{u'(r)}{r} = 1,$$

with u(0) = u'(0) = 0. Moreover, as  $r \to \infty$ 

$$u(r) = \frac{r^2}{2(n-1)} - \ln r + O(r^{-1}),$$

that is, U = Graph(u) is asymptotic to a parabola.

We note that Clutterbuck, Schnürer and Schulze [3] showed that there exist non-convex rotationally symmetric translating solitons in  $\mathbb{R}^3$  named *winglike* translating solitons. Nguyen [14,15] constructed various complete embedded translating solitons in  $\mathbb{R}^3$  using a gluing technique.

Now we consider compact translating solitons with non-empty planar boundary. From the view point of rotationally symmetric surfaces, we show that the compact part of U separated by a horizontal plane is characterized as follows:

**Main Theorem.** Let U be the entire rotationally symmetric strictly convex graphical translating soliton in  $\mathbb{R}^3$ . Let  $\Sigma$  be a compact connected immersed translating soliton with smooth boundary  $\Gamma = \partial \Sigma$  lying in a horizontal plane  $\Pi = \{(x, y, z) | z = c \in \mathbb{R}\}.$ 

- (1) If  $\Gamma$  is a circle, then  $\Sigma$  is part of U.
- (2) If  $\Gamma$  is convex and  $\Sigma$  meets  $\Pi$  with a constant angle along  $\Gamma$ , then  $\Sigma$  is also part of U.

When a compact translating soliton  $\Sigma$  is graphical over a domain D in a horizontal plane, then there exists a function u(x, y) so that  $Graph(u) = \Sigma$  and it satisfies

$$\sqrt{1+|\nabla u|^2}div\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right) = 1,$$

for all  $(x, y) \in D$ . Serrin [17, §20] showed that the necessary and sufficient conditions for existence of such a function are that the domain D is mean convex. So there are plenty of compact translating solitons with planar boundary which are different from a compact part of U separated by a horizontal plane. We note that Serrin's result holds for higher dimension.

### 2. Proof of the Main Theorem

A fundamental tool to prove the Main Theorem is the tangency principle based on maximum principle of elliptic partial differential equations. Download English Version:

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