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Weak solutions of the Landau–Lifshitz–Bloch equation ☆

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Abstract

The Landau–Lifshitz–Bloch (LLB) equation is a formulation of dynamic micromagnetics valid at all temperatures, treating both the transverse and longitudinal relaxation components important for high-temperature applications. We study LLB equation in case the temperature raised higher than the Curie temperature. The existence of weak solution is showed and its regularity properties are also discussed. In this way, we lay foundations for the rigorous theory of LLB equation that is currently not available.

MSC: 82D40; 35K59; 35R15

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1. Introduction

Micromagnetic modeling has proved itself as a widely used tool, complimentary in many respects to experimental measurements. The Landau–Lifshitz–Gilbert (LLG) equation [21,16] provides a basis for this modeling, especially where the dynamical behavior is concerned. According to this theory, at temperatures below the critical (so-called Curie) temperature, the magnetization $\boldsymbol{m}(t,\boldsymbol{x}) \in \mathbb{S}^2$, where $\mathbb{S}^2$ is the unit sphere in $\mathbb{R}^3$, for $t > 0$ and $\boldsymbol{x} \in D \subset \mathbb{R}^d$, $d = 1, 2, 3$, satisfies the following LLG equation

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$$\frac{\partial \boldsymbol{m}}{\partial t} = \lambda_1 \boldsymbol{m} \times \boldsymbol{H}_{\text{eff}} - \lambda_2 \boldsymbol{m} \times (\boldsymbol{m} \times \boldsymbol{H}_{\text{eff}}), \tag{1.1}$$

where $\times$ is the vector cross product in $\mathbb{R}^3$ and $\boldsymbol{H}_{\text{eff}}$ is the so-called effective field.

However, for high temperatures the model must be replaced by a more thermodynamically consistent approach such as the Landau–Lifshitz–Bloch (LLB) equation [14,15]. The LLB equation essentially interpolates between the LLG equation at low temperatures and the Ginzburg–Landau theory of phase transitions. It is valid not only below but also above the Curie temperature T_c. An important property of the LLB equation is that the magnetization magnitude is no longer conserved but is a dynamical variable [15,11]. The spin polarization $\boldsymbol{u}(t,\boldsymbol{x}) \in \mathbb{R}^3$, ($\boldsymbol{u} = \boldsymbol{m}/m_s^0$, $\boldsymbol{m}$ is magnetization and m_s^0 is the saturation magnetization value at $T = 0$), for $t > 0$ and $\boldsymbol{x} \in D \subset \mathbb{R}^d$, $d = 1, 2, 3$, satisfies the following LLB equation

$$\frac{\partial \boldsymbol{u}}{\partial t} = \gamma \boldsymbol{u} \times \boldsymbol{H}_{\text{eff}} + L_1 \frac{1}{|\boldsymbol{u}|^2}(\boldsymbol{u} \cdot \boldsymbol{H}_{\text{eff}})\boldsymbol{u} - L_2 \frac{1}{|\boldsymbol{u}|^2} \boldsymbol{u} \times (\boldsymbol{u} \times \boldsymbol{H}_{\text{eff}}). \tag{1.2}$$

Here, $|\cdot|$ is the Euclidean norm in $\mathbb{R}^3$, $\gamma > 0$ is the gyromagnetic ratio, and L_1 and L_2 are the longitudinal and transverse damping parameters, respectively.

LLB micromagnetics has become a real alternative to LLG micromagnetics for temperatures which are close to the Curie temperature ($T \gtrsim \frac{3}{4} T_c$). This is realistic for some novel exciting phenomena, such as light-induced demagnetization with powerful femtosecond (fs) lasers [2]. During this process the electronic temperature is normally raised higher than T_c. Micromagnetics based on the LLG equation cannot work under these circumstances while micromagnetics based on the LLB equation has proved to describe correctly the observed fs magnetization dynamics.

In this paper, we consider a deterministic form of a ferromagnetic LLB equation, in which the temperature T is raised higher than T_c, and as a consequence the longitudinal L_1 and transverse L_2 damping parameters are equal. The effective field $\boldsymbol{H}_{\text{eff}}$ is given by

$$\boldsymbol{H}_{\text{eff}} = \Delta \boldsymbol{u} - \frac{1}{\chi_{||}}\left(1 + \frac{3}{5}\frac{T}{T - T_c}|\boldsymbol{u}|^2\right)\boldsymbol{u},$$

where $\chi_{||}$ is the longitudinal susceptibility.

By using the vector triple product identity $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b}(\boldsymbol{a} \cdot \boldsymbol{c}) - \boldsymbol{c}(\boldsymbol{a} \cdot \boldsymbol{b})$, we get

$$\boldsymbol{u} \times (\boldsymbol{u} \times \boldsymbol{H}_{\text{eff}}) = (\boldsymbol{u} \cdot \boldsymbol{H}_{\text{eff}})\boldsymbol{u} - |\boldsymbol{u}|^2 \boldsymbol{H}_{\text{eff}},$$

and from property $L_1 = L_2 =: \kappa_1$, we can rewrite (1.2) as follows

$$\frac{\partial \boldsymbol{u}}{\partial t} = \kappa_1 \Delta \boldsymbol{u} + \gamma \boldsymbol{u} \times \Delta \boldsymbol{u} - \kappa_2(1 + \mu|\boldsymbol{u}|^2)\boldsymbol{u}, \quad \text{with } \kappa_2 := \frac{\kappa_1}{\chi_{||}}, \quad \mu := \frac{3T}{5(T - T_c)}. \tag{1.3}$$

So the LLB equation we are going to study in this paper is equation (1.3) with real positive coefficients $\kappa_1, \kappa_2, \gamma, \mu$, initial data $\boldsymbol{u}(0,\boldsymbol{x}) = \boldsymbol{u}_0(\boldsymbol{x})$ and subject to homogeneous Neumann boundary conditions.

Various results on existence of global weak solutions of the LLG equation (1.1) are proved in [8,1]. More complete lists can be found in [9,18,20]. Furthermore, there is also some research