J. Math. Anal. Appl. 446 (2017) 307–321

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

# Global regularity for a class of 2D tropical climate model

Zhuan Ye

*Department of Mathematics and Statistics, Jiangsu Normal University, 101 Shanghai Road, Xuzhou 221116, Jiangsu, PR China*

ARTICLE INFO

*Article history:*
Received 22 April 2016
Available online 1 September 2016
Submitted by P.G. Lemarie-Rieusset

*Keywords:*
Tropical atmospheric dynamics
Global regularity

ABSTRACT

In this paper, we establish the global regularity of a modified version of the tropical climate model originally derived by Frierson–Majda–Pauluis (2004) [6] which couples the barotropic and baroclinic modes of the velocity and the typical midtropospheric temperature. In proving the global regularity, to bypass the difficulty caused by the very weak dissipation (any small power $\alpha > 0$) of the barotropic mode, we utilize the "weakly nonlinear" energy estimate approach and the Maximal $L_t^q L_x^p$ regularity for the heat kernel.

© 2016 Elsevier Inc. All rights reserved.

## 1. Introduction

This paper focuses on the initial-value problem for the following two dimensional (2D) tropical climate model originally introduced by Frierson–Majda–Pauluis in [6]

$$\begin{cases} \partial_t u + (u\cdot\nabla)u + \mu\Lambda^{2\alpha}u + \nabla p + \nabla\cdot(v\otimes v) = 0, \quad x\in\mathbb{R}^2,\ t>0, \\ \partial_t v + (u\cdot\nabla)v - \nu\Delta v + \nabla\theta + (v\cdot\nabla)u = 0, \\ \partial_t\theta + (u\cdot\nabla)\theta - \eta\Delta\theta + \nabla\cdot v = 0, \\ \nabla\cdot u = 0, \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad \theta(x,0) = \theta_0(x), \end{cases} \tag{1.1}$$

where $u = (u_1(x,t), u_2(x,t))$ is the barotropic mode, $v = (v_1(x,t), v_2(x,t))$ is the first baroclinic mode of the vector velocity, $p = p(x,t)$ is the scalar pressure, $\theta = \theta(x,t)$ is scalar temperature, respectively. The $\alpha \in (0,1)$ is the fractional power (in fact, our main focus is $\alpha > 0$ arbitrarily small) and the fractional Laplacian operator $\Lambda^{2\alpha}$ is defined through the Fourier transform, namely $\widehat{\Lambda^{2\alpha}f}(\xi) = |\xi|^{2\alpha}\widehat{f}(\xi)$. The parameters $\mu$, $\nu$ and $\eta$ are non-negative. The $(i,j)$ component of matrix $v\otimes v$ is $v_i v_j$. For more details of the related background about the tropical climate model, for instance, see [7,16,17] and the references therein.

*E-mail address:* yezhuan815@126.com.

http://dx.doi.org/10.1016/j.jmaa.2016.08.053
0022-247X/© 2016 Elsevier Inc. All rights reserved.

Let us first briefly review some results of the tropical climate model (1.1). By performing a Galerkin truncation to the hydrostatic Boussinesq equations (see [6, Section 2]), Frierson–Majda–Pauluis derived the original version of (1.1) without any dissipation terms (namely $\mu = \nu = \eta = 0$). Very recently, by introducing a new quantity and using a Logarithmic Gronwall inequality, Li and Titi [15] proved the global well-posedness of strong solutions for the system (1.1) without diffusivity in the temperature equation, namely, $\alpha = 1$ ($\mu > 0$), $\nu > 0$ and $\eta = 0$. Later, inspired by [15], Wan [19] obtained the global well-posedness of classical solutions to a tropical climate model with the dissipation of the first baroclinic model of the velocity and some damping terms under small initial data.

Here it is worth particularly mentioning the following one closely related system-2D magnetohydrodynamic (MHD) equations

$$\begin{cases} \partial_t u + (u\cdot\nabla)u + \mu\Lambda^{2\alpha}u + \nabla p - (b\cdot\nabla)b = 0, \quad x\in\mathbb{R}^2,\ t>0, \\ \partial_t b + (u\cdot\nabla)b - \nu\Delta b - (b\cdot\nabla)u = 0, \\ \nabla\cdot u = 0, \quad \nabla\cdot b = 0, \\ u(x,0) = u_0(x),\ b(x,0) = b_0(x), \end{cases} \tag{1.2}$$

where $u = (u_1(x,t), u_2(x,t))$, $p = p(x,t)$ and $b = (b_1(x,t), b_2(x,t))$ denote the velocity vector, scalar pressure, and the magnetic field of the fluid, respectively. The 2D MHD equations attracted quite a lot of attention lately from various authors. Actually, there is a considerable body of literature on the global regularity of the 2D MHD equations. We recall here, without any claim of completeness, see [4,5,8,9,18,23, 20–22,24] and the references cited therein. Obviously, when $\theta =$ constant, the system (1.1) reduces to the 2D MHD-type equations (1.2) (regard $v$ as $b$).

Now let us compare the tropical climate model (1.1) with the MHD equations (1.2) and explain the difficulty as well as our approach. As a matter of fact, it is much harder to get the global regularity result for system (1.1) than for that of the system (1.2). The main reason is that it is easy to get the global $H^1$-estimation for the system (1.2) by energy estimate (see [5,18,22]). Unfortunately, we are unable to obtain the global $H^1$-estimation for the system (1.1) via the general energy method. The main difficulty comes from the following term (without $\nabla\cdot v = 0$)

$$\sum_{i,j,l=1}^{2}\int_{\mathbb{R}^2}\partial_l\partial_i v_i v_j \partial_l u_j\,dx \quad \text{or} \quad \sum_{j,l=1}^{2}\int_{\mathbb{R}^2}\partial_l(\nabla\cdot v)v_j\partial_l u_j\,dx, \quad \text{see (2.7) for details.} \tag{1.3}$$

However, for the system (1.2), we do not need to face that problem (due to $\nabla\cdot b = 0$), and the global regularity issue has been solved by [5]. Therefore, compared with the system (1.2), our result is more involved in the proof. Indeed, it is not clear how to show the global regularity via the direct energy method. Fortunately, to overcome the main obstacle caused by the term in (1.3), we make use of the "weakly nonlinear" energy estimate approach, which enables us to get the following "almost a priori" $H^1$-estimation

$$\begin{aligned} &\|\nabla u(t)\|_{L^2}^2 + \|\nabla v(t)\|_{L^2}^2 + \|\nabla\theta(t)\|_{L^2}^2 + \int_{T_0}^{t}(\|\Lambda^\alpha\nabla u\|_{L^2}^2 + \|\Delta v\|_{L^2}^2 + \|\Delta\theta\|_{L^2}^2)(\tau)\,d\tau \\ &\le C(1+M(t))^\epsilon, \quad \text{for any } T_0 \le t < T, \end{aligned} \tag{1.4}$$

where

$$M(t) := \max_{\tau\in[T_0,\,t]}\|\nabla v(\tau)\|_{L^q}, \quad \text{for some } \ 2 < q \le \frac{4}{2-\alpha}.$$

Download English Version:

https://daneshyari.com/en/article/4613817

Download Persian Version:

https://daneshyari.com/article/4613817

Daneshyari.com