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Uniform stability of the solutions to the relativistic Enskog equation

Jianjun Huang, Zhenglu Jiang *

Department of Mathematics, Sun Yat-Sen University, Guangzhou 510275, China

A R T I C L E I N F O

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A B S T R A C T

We present the uniform stability of classical solutions to the relativistic Enskog equation under the locally Lipschitz assumption of the collision factors given by Polewczak. This stability is an extension of the result given by Ha and Xiao for the relativistic Boltzmann equation and it is very useful for the study of the Cauchy problem for the relativistic Enskog equation.

1. Introduction

We are concerned with the uniform stability of the solutions to the relativistic Enskog equation for the moderately or highly dense relativistic gases. The relativistic Enskog equation [24] can be written in the following form:

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{p_0}\frac{\partial f}{\partial \mathbf{x}} = Q(f, f), \tag{1.1}$$

where $f = f(t, \mathbf{x}, \mathbf{p})$ denotes a distribution function of a one-particle relativistic gas with the momentum $\mathbf{p} \in \mathbf{R}^3$ at the space position $\mathbf{x} \in \mathbf{R}^3$ and the time $t \in (0, \infty)$; $p_0 = (1 + |\mathbf{p}|^2)^{1/2}$ is the energy of a dimensionless relativistic gas particle with the momentum $\mathbf{p}$; $Q(f, f)$ is the relativistic Enskog collision operator describing the binary collision with the difference between the gain and loss terms:

$$Q(f, f) = Q^+(f, f) - Q^-(f, f). \tag{1.2}$$

In order to show the two terms in (1.2), we have to use $(\mathbf{p}, \mathbf{p}_*)$ and $(\mathbf{p}', \mathbf{p}'_*)$ to represent the momenta of two relativistic particles immediately before collision and after collision, respectively. All the relations between

* Corresponding author.
E-mail addresses: mcshjj@hotmail.com (J. Huang), mcsjzl@mail.sysu.edu.cn (Z. Jiang).

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the momenta $(\mathbf{p}, \mathbf{p}_*)$ and $(\mathbf{p}', \mathbf{p}'_*)$ are the same as obtained for the relativistic Boltzmann equation [8]. Moreover, let p_{*0} denote the dimensionless energy of the colliding relativistic gas particle with the momentum $\mathbf{p}_*$ before collision, then $p_{*0} = (1 + |\mathbf{p}_*|^2)^{1/2}$. As used below in the same way, put $p'_0 = (1 + |\mathbf{p}'|^2)^{1/2}$ and $p'_{*0} = (1 + |\mathbf{p}'_*|^2)^{1/2}$, then we know that they are respectively the dimensionless energy of the two relativistic particles after collision. Thus the gain and loss terms can be expressed as follows [13]:

$$Q^+(f,f) = a^2 \int\limits_{R^3 \times S^2_+} F^+(f) \frac{gs^{1/2}}{p_0 p_{*0}} \sigma(g,\theta) f(t,\mathbf{x},\mathbf{p}') f(t,\mathbf{x} + a\omega, \mathbf{p}'_*) d\omega d\mathbf{p}_*, \tag{1.3}$$

$$Q^-(f,f) = a^2 \int\limits_{R^3 \times S^2_+} F^-(f) \frac{gs^{1/2}}{p_0 p_{*0}} \sigma(g,\theta) f(t,\mathbf{x},\mathbf{p}) f(t,\mathbf{x} - a\omega, \mathbf{p}_*) d\omega d\mathbf{p}_*, \tag{1.4}$$

where a is the diameter of hard sphere, of course, $a > 0$, and the collision factors $F^{\pm}(f)$ are two nonnegative functionals of f. By the way, the derivation of (1.1) is analogous to that of the relativistic Boltzmann equation. The other different parts in (1.3) and (1.4) are explained as follows.

$\mathbf{R}^3$ is a three-dimensional Euclidean space and $S^2_+ = \{\omega \in S^2 : \omega(\mathbf{p}/p_0 - \mathbf{p}_*/p_{*0}) \geq 0\}$ is a subset of the unit sphere surface S^2 with an infinitesimal element $d\omega = \sin\theta d\theta d\varphi$ for the scattering angle $\theta \in [0, \pi]$ and the other solid angle $\varphi \in [0, 2\pi]$ in the center-of-momentum system [9,10]. $s = |p_{*0} + p_0|^2 - |\mathbf{p}_* + \mathbf{p}|^2$ and $s^{1/2}$ is the total energy in the center-of-mass frame; $g = \sqrt{|\mathbf{p}_* - \mathbf{p}|^2 - |p_{*0} - p_0|^2}/2$ and $2g$ is in fact the value of the relative momentum in the center-of-mass frame [9,10]; it can be seen that $s = 4 + 4g^2$. $\sigma(g, \theta)$ is the differential scattering cross section of the variable g and the scattering angle θ, and the scattering angle θ is defined by

$$\cos\theta = 1 - 2[(p_0 - p_{*0})(p_0 - p'_0) - (\mathbf{p} - \mathbf{p}_*)(\mathbf{p} - \mathbf{p}')]/(4 - s).$$

It is worth mentioning that $\frac{gs^{1/2}}{p_0 p_{*0}}$ is equal to the Moller velocity [16] defined by

$$v_m = \sqrt{|\mathbf{p}/p_0 - \mathbf{p}_*/p_{*0}|^2 - |\mathbf{p}/p_0 \times \mathbf{p}_*/p_{*0}|^2},$$

which is bounded above by $|\mathbf{p}/p_0 - \mathbf{p}_*/p_{*0}|$. Also, some estimates of s and g have been given by Glassey and Strauss [16] and they can be used for the relativistic Enskog equation as well.

The Enskog equation was derived by Enskog [11] in 1992 to describe the moderately or highly dense gases. It can be seen as a modification of the Boltzmann equation when the density of the gas increases. Compared with the Boltzmann equation, the Enskog equation has the extra factors $F^{\pm}$ in the collision operator. In general, these factors are the functionals of the density. Moreover, the collision described by the Enskog equation is not considered to take place at the same position for two gas particles. As we know, the associated collision positions are $\mathbf{x}$ and $\mathbf{x} + a\omega$ in (1.3), but they are $\mathbf{x}$ and $\mathbf{x} - a\omega$ in (1.4). Arkery and Cercignani [4] showed that in the classical case, if the collision factors are the same constant, then the solution of the Enskog equation converges to the solution of the related Boltzmann equation as the diameter a tends to zero. It can be proved that there is a similar result between the solutions of the relativistic Enskog and Boltzmann equations. Hence the relativistic Enskog equation can also be seen as a modification of the relativistic Boltzmann equation. The background information of the relativistic Boltzmann equation can be found in the work of the previous researchers, such as Andreasson [1], Bichteler [5], Glassey [14], Glassey and Strauss [15–17], Jiang [21–23,25] and Strain [33–35].

Now we review some previous work on the Enskog equation in both classical and relativistic cases. In the classical case, Lachowicz [29] first proved in 1983 that the Enskog equation admits a unique local solution. Polewczak [30] showed in 1989 that there exists a global solution to the Enskog equation under the assumption that the initial data are near vacuum data and that the collision factors satisfy the locally