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Corrigendum

Corrigendum to "Singular integral operators along surfaces of revolution" [J. Math. Anal. Appl. 274 (2) (2002) 608–625]

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ABSTRACT

One of the key estimates in the original paper [3] does not seem to hold true, which may invalidate some results of that paper. However, there is a couple of ways to fix this mistake by adding additional hypotheses in Theorem 1 [3].

In the proof of Theorem 1 [3], both inequalities

$$|\widehat{\sigma}_k(\zeta, \zeta_{n+1})| \le C \left|2^{dk} A_\rho \zeta\right| \tag{0.1}$$

and

$$|\widehat{\mu}_k(\zeta, \zeta_{n+1}) - \widehat{\mu}_k(0, \zeta_{n+1})| \le C|2^{kd} A_\rho \zeta| \tag{0.2}$$

rely on the estimate

$$\int_{\zeta_1'-3r}^{\zeta_1'+3r} |s|\, ds \le C\, r^2, \quad \text{where} \quad r \equiv r(\zeta') = \rho\sqrt{(\rho\zeta_1')^2 + (\zeta_2')^2 + \cdots + (\zeta_n')^2}; \;\; (\zeta_1', ..., \zeta_n') \in S^{n-1}. \tag{0.3}$$

Unfortunately, inequality (0.3) may not hold true unless $|\zeta_1'| \le C\, r(\zeta')$.

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Inequality (0.1) still holds true by properly using the cancellation property of $F_a(s,\zeta')$ as follows:

$$
\begin{aligned}
|\widehat{\sigma}_k(\zeta,\zeta_{n+1})| &= \left| \int_{2^k}^{2^{k+1}} \frac{h(t)}{t}\, e^{i\zeta_{n+1}\gamma(t)} \int \left(e^{i|\zeta|\phi(t)s} - e^{i|\zeta|\phi(t)\zeta_1'}\right) F_a(s,\zeta')\, ds\, dt \right| \\
&\leq \left\{ \int_{2^k}^{2^{k+1}} |\zeta|\; |h(t)\phi(t)|\, \frac{dt}{t} \right\} \left\{ \int |(s-\zeta_1')\, F_a(s,\zeta')|\; ds \right\} \\
&\leq C\left|2^{dk}\zeta\right| r^{-1} \int_{\zeta_1'-3r}^{\zeta_1'+3r} |s-\zeta_1'|\, ds \\
&\leq C\left|2^{dk} r\zeta\right| = C\left|2^{dk}A_\rho\zeta\right|.
\end{aligned}
$$

On the other hand, it seems to be impossible to obtain inequality (0.2) due to the presence of ζ_1'. Note that inequality (0.2) was needed in [3] to obtain the L^p-boundedness $(1<p<\infty)$ of $\sup_{k\in\mathbb{Z}}|\mu_k * f|$ via Theorem C* in [3]. We can still obtain the L^p-boundedness of $\sup_{k\in\mathbb{Z}}|\mu_k * f|$ by a different technique and by requiring additional hypotheses on γ. We now proceed its proof below.

Recall from [3] that

$$
\begin{aligned}
(\mu_k * f)(x,\, x_{n+1}) &= \int_{|y|\cong 2^k} \frac{|h(|y|)\, a(y')|}{|y|^n} f\left(x-\phi(|y|)y',\, x_{n+1}-\gamma(|y|)\right)\, dy \\
&\leq C \int_{|y|\cong 2^k} \frac{|a(y')|}{|y|^n} f\left(x-\phi(|y|)y',\, x_{n+1}-\gamma(|y|)\right)\, dy \\
&\equiv C\,(\nu_k * f)(x,\, x_{n+1}),
\end{aligned}
$$

where the inequality follows from the assumption that the function h is bounded almost everywhere. Thus, it suffices to prove that $\sup_{k\in\mathbb{Z}}|\nu_k * f|$ is bounded on $L^p(\mathbb{R}^{n+1})$ for $1<p<\infty$. By the method of rotation, it is enough to prove the L^p-boundedness $(1<p<\infty)$ of the following two-dimensional maximal function

$$
Mg(x_1,\, x_2) = \sup_{k\in\mathbb{Z}} \left\{ \frac{1}{2^k} \int_{2^k}^{2^{k+1}} |g\left(x_1-\phi(t),\, x_2-\gamma(t)\right)|\, dt \right\}.
$$

We will apply Theorem C [1] to prove. We may assume that $g\geq 0$. Define the positive, finite Borel measures $\{\lambda_k\}_{k\in\mathbb{Z}}$ as

$$
(\lambda_k * g)(x_1,\, x_2) = \frac{1}{2^k} \int_{2^k}^{2^{k+1}} g\left(x_1-\phi(t),\, x_2-\gamma(t)\right)\, dt.
$$

In terms of Fourier transform,

$$
\hat{\lambda}_k(\zeta_1,\, \zeta_2) = \frac{1}{2^k} \int_{2^k}^{2^{k+1}} e^{i\zeta_1\phi(t)}\, e^{i\zeta_2\gamma(t)}\, dt.
$$