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A new regularity criterion for the 3D generalized Hall-MHD system with $\beta \in (\frac{1}{2}, 1]$

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In this paper, we consider the generalized Hall-MHD system in dimension three. A new regularity criterion is established for $\beta \in (\frac{1}{2}, 1]$.

1. Introduction

In this paper, we consider the following 3D generalized Hall-MHD system:

$$u_t + u \cdot \nabla u + \Lambda^{2\alpha} u + \nabla P = (\nabla \times b) \times b, \tag{1.1}$$

$$b_t - \nabla \times (u \times b) + \nabla \times ((\nabla \times b) \times b) + \Lambda^{2\beta} b = 0, \tag{1.2}$$

$$\operatorname{div} u = \operatorname{div} b = 0, \tag{1.3}$$

here $u = u(x,t) \in \mathbb{R}^3$, $b = b(x,t) \in \mathbb{R}^3$, $p = p(x,t) \in \mathbb{R}$ represent the unknown velocity field, the magnetic field and the pressure, respectively. $\alpha > 0$, $\beta > 0$ are real parameters. A fractional power of the Laplace transform, $\Lambda^{2\alpha}$ is defined through the Fourier transform

$$\widehat{\Lambda^{2\alpha} f}(\xi) = |\xi|^{2\alpha} \widehat{f}(\xi).$$

The local well-posedness is established by Chae, Wan and Wu in [6] for the case $\mu = 0$ and $\beta > \frac{1}{2}$. Very recently, Wan and Zhou [26] established the local well-posedness for the case $\alpha \in (0, 1]$ and $\beta \in (\frac{1}{2}, 1]$. More precisely, they proved that

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Theorem 1.1. *([26]) Consider (1.1)–(1.3) with* $\alpha \in (0, 1]$ *and* $\beta \in (\frac{1}{2}, 1]$. *Assume* $u_0, b_0 \in H^s(\mathbb{R}^3)$ *with* $s > \max\{\frac{5}{2} - 2\alpha, \frac{7}{2} - 2\beta\}$. *Then there exists* $T_0 = T_0(\|u_0\|_{H^s}, \|b_0\|_{H^s})$ *and a unique solution* $(u; b)$ *of (1.1)–(1.3) on* $[0; T_0]$ *such that*

$$(u; b) \in C(0; T_0; H^s(\mathbb{R}^3)).$$

Moreover,

$$\int_0^{T_0} \|\Lambda^\alpha u\|_{H^s}^2 + \|\Lambda^\beta b\|_{H^s}^2 \, dt \le C(T_0, \|u_0\|_{H^s}, \|b_0\|_{H^s}).$$

In [17], they established the regularity criterion for the case $\frac{5}{4} > \alpha \ge \frac{3}{4}$, $\frac{7}{4} > \beta \ge 1$. They proved that if

$$\nabla B \in L^t(0, T, L^s(\mathbb{R}^3)) \ \text{ with } \ \frac{2\beta}{t} + \frac{3}{s} \le 2\beta - 1, \ \ \frac{3}{2\beta - 1} < q \le \infty$$

and one of the following two conditions

$$u \in L^p(0, T, L^q(\mathbb{R}^3)) \ \text{ with } \ \frac{2\alpha}{p} + \frac{3}{q} \le 2\alpha - 1, \ \ \frac{3}{2\alpha - 1} < q \le \frac{6\alpha}{2\alpha - 1},$$

or

$$\Lambda^\alpha u \in L^p(0, T, L^q(\mathbb{R}^3)) \ \text{ with } \ \frac{2\alpha}{p} + \frac{3}{q} \le 3\alpha - 1, \ \ \frac{3}{2\alpha - 1} < q \le \frac{6\alpha}{3\alpha - 1},$$

then the solution remains smooth on $[0, T]$. Some other criteria can be found in [27] where $\beta \ge 1$ is also needed.

When $\alpha = \beta = 1$, the generalized Hall-MHD system reduce to the following Hall-MHD system which is studied by M. J. Lighthill [20]

$$u_t + u \cdot \nabla u - \Delta u + \nabla P = (\nabla \times b) \times b, \tag{1.4}$$

$$b_t - \nabla \times (u \times b) + \nabla \times ((\nabla \times b) \times b) - \Delta b = 0, \tag{1.5}$$

$$\operatorname{div} u = \operatorname{div} b = 0. \tag{1.6}$$

The Hall-MHD system is useful in describing many physical phenomena in geophysics and astrophysics. Mathematical derivations of Hall-MHD equations from either two-fluids or kinetic models can be found in [1]. The existence of global weak solutions in the case of a periodic domain been proved in [1] by using a Galerkin approximation. Chae and his collaborators [2] got the existence of global weak solutions in the whole space case. They also got the local existence and uniqueness of smooth solutions in [2]. Later, the Serrin type criterion

$$u \in L^p(0, T; L^q), \ \nabla b \in L^t(0, T; L^s) \ \text{ with } \ 2/p + 3/q \le 1 \ \text{ and } \ 2/t + 3/s \le 1$$

was proved by Chae and Lee in [3]. They also got the criterion in the BMO space

$$(u, \ \nabla b) \in L^2(0, T; BMO).$$

Several regularity criteria which were established in the Besov space, BMO space, Lorenz space can be found in [8–16,24,25,28]. The temporal decay and singularity formation are investigated in [4,5]. In [25], they proved