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On the detectability and observability of continuous stochastic Markov jump linear systems

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ABSTRACT

This paper is concerned with the detectability and observability of continuous linear stochastic systems subject to Markov jump. By introducing some operators, we investigate the relationship between some different concepts of detectability and observability, and obtain their sufficient and necessary conditions. Continuous-time stochastic Lyapunov equations and stochastic continuous algebraic Riccati equations are also discussed to serve as analysis tools.

As a result of its widespread use in many disciplines, stochastic differential equations have been widely investigated during the past decades, e.g., see [16,22]. Another characteristic of practical world is abrupt changes [3,9,13,21]. Among the models subject to abrupt changes, Markov jump linear systems have attracted extensive attention [3,5,9,23].

During the past decades, the study of linear stochastic systems is always parallel to that of linear deterministic systems in many ways [14,23]. For example, mean square stability can be viewed as the stochastic version of asymptotic stability in deterministic systems [3]. And stabilizability of stochastic linear systems plays the same role to guarantee existence, uniqueness of closed-loop solutions of quadratic infinite horizon control problems (see, e.g., [9] and [3]). More often than not stochastic systems have their own distinguishable properties, and some definitions derived analogously from deterministic systems are too strong. For example, detectability was defined as dual to the mean square stabilization (see, e.g., [2]). This definition was based on the property that all unstable modes produce some non-zero output, which is found to be only a necessary but not a sufficient condition for stabilization of the dual system in the stochastic setting [6,17]. To fill this gap, some weaker concepts of detectability were proposed, respectively, for discrete and continuous Markov jump linear systems, see, e.g., [1,3,15] and the references therein. And Hautus criteria are also provided for the weaker detectability based on the spectral technique [15].

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In some literature observability is always defined in the same framework of detectability [3,11,12,15]. For exact observability one can refer to [12,18,19] for equivalent but different forms. Another weaker observability definition was given in [8].

We can find that these detectability and observability concepts for stochastic systems are defined in different forms. That motivates us to understand the links between them. Meanwhile, for all we know, so far there are very few results on the problem of the observability and detectability of continuous linear stochastic systems subject to Markov jump.

Motivated by the discussions above, in this paper we will investigate the detectability and observability of continuous linear stochastic systems subject to Markov jump. We begin with studying the second moment of the state $x(t)$ and its evolution dynamics. Then we define the observability Gramian and study its properties to establish the relations between different concepts of detectability and observability of continuous linear stochastic systems subject to Markov jump. These results can help us obtain some necessary and sufficient conditions such as Haust-test of detectability and observability of continuous linear stochastic systems subject to Markov jump. Furthermore, continuous-time stochastic Lyapunov equations and stochastic continuous algebraic Riccati equations are also discussed to serve as analysis tools. It is particularly pointed out that even our results can be viewed as the continuous version of [17], the results are not trivial due to different settings and techniques.

This paper is organized as follows. In Section 1, we recall some preliminary facts. Section 2 will give main results about detectability and observability. And concluding remarks are presented in Section 3.

1. Preliminaries

Throughout this paper, we use $\mathcal{R}^n \subseteq \mathbb{R}^{n\times n}$ to represent the normed linear space of $n \times n$ real matrices. The notation $\mathcal{S}^{n,N}$ denotes the linear space $\mathcal{S}^{n,N} = \{H = (H_1, \dots, H_N), H_i \in \mathcal{R}^n, i = 1, 2, \dots, N\}$. $\mathcal{S}^{n,N}$ can be organized as a Hilbert space by the following inner product [17]

$$\langle U, V\rangle = \sum_{k=1}^{N} \operatorname{tr}\big(U_k^T V_k\big) \tag{1}$$

for all $U = (U_1, \dots, U_N)$ and $V = (V_1, \dots, V_N) \in \mathcal{S}^{n,N}$.

We consider the following continuous-time linear stochastic systems with Markov jump

$$\begin{aligned} \mathrm{d}x(t) &= A_0\big(r(t)\big)x(t)\mathrm{d}t + \sum_{p=1}^{l} A_p\big(r(t)\big)x(t)\mathrm{d}w_p(t), \\ y(t) &= \left[C_0\big(r(t)\big) + \sum_{p=1}^{l} C_p\big(r(t)\big)w_p(t)\right]x(t), \end{aligned} \tag{2}$$

defined in a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$, with initial data $x(0) = x_0$, $r(0) \sim \mu_0$. Here $x(t) \in \mathbb{R}^n$ and $y(t) \in \mathbb{R}^m$ are respectively the state and the measured output. The node r is the state of homogeneous continuous-time Markov chain $\{r(t), t \geq 0\}$ having $D = \{1, 2, \dots, N\}$ as state space and $\mathbb{P} = (p_{ij})$, $i, j = 1, \dots, N$ as the transition rate matrix. The initial distribution of r is determined by $\mu_i = P(r_0 = i)$, $i = 1, 2, \dots, N$. $w_p(t) \in \mathbb{R}$, $p = 1, \dots, l$ are sequences of independent white noise with $\mathbb{E}(w_i(t)w_j(t)) = \delta_{ij}$, $\mathbb{E}(w_i(t)) = 0$, $i, j = 1, \dots, l$. The Markov chain $r(t)$ is assumed to be independent of $\{w(t) = (w_1(t), ..., w_l(t))^T, t \geq 0\}$. Set $A = (A_0, A_1, \dots, A_l)$, $C = (C_0, C_1, \dots, C_l)$ where A_p stands for the sequence $A_p(i) \in \mathbb{R}^{n\times n}$, $i \in D$, $p = 0, 1, \dots, l$. $\dim(C_p) = m \times n$.

In order to obtain differential dynamics that the second moments of $x(t)I_{\{r(t)=i\}}$ satisfies, we introduce the following operators.