J. Math. Anal. Appl. 417 (2014) 694–718

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Optimal control for infinite dimensional stochastic differential equations with infinite Markov jumps and multiplicative noise

Viorica Mariela Ungureanu

Department of Mathematics, "Constantin Brancusi" University, Tg. Jiu, Bulevardul Republicii, nr. 1, jud. Gorj, Romania

ARTICLE INFO

Article history:
Received 20 August 2013
Available online 25 March 2014
Submitted by Goong Chen

Keywords:
Stochastic differential equations
Linear quadratic control
Generalized Riccati differential equations
Detectability
Ito's formula

ABSTRACT

In this paper we solve an infinite-horizon linear quadratic control problem for a class of differential equations with countably infinite Markov jumps and multiplicative noise. The global solvability of the associated differential Riccati-type equations is studied under detectability hypotheses. A nonstochastic, operatorial approach is used. Some properties of the linear stochastic systems, such as stability, stabilizability and detectability, are also discussed on the basis of a new solution representation result. A generalized Ito's formula which applies to infinite dimensional stochastic differential equations with countably infinite Markov jumps is also provided.

1. Introduction

Stochastic differential equations (SDEs) with Markovian switching can model many physical systems which may experience abrupt changes in their dynamics. Among them we mention the manufacturing systems, the power systems, the telecommunication systems etc. All these systems suffer frequent unpredictable structural changes caused by failures or repairs, connections or disconnections of the subsystems [2]. The new applications in modern queuing network theory or in the field of safety-critical and high integrity systems [3,8] led to a revival of the study of SDEs with Markovian jumps (MJs), especially in the case where the state space of the Markov process is countably infinite. For a sample of works dealing with stability, optimal control and $\mathcal{H}_\infty$ stabilization problems for SDEs with MJs see [19,2,5,6,4,11,8,9] and the references therein.

First results on optimal control problems for linear SDEs with MJs and countably infinite state space for the Markov process (SDEs with infinite MJs) were obtained recently in [8–10].

Unlike these works, we consider in this paper an optimal control problem for time-varying linear SDEs with infinite MJs and multiplicative noise (MN) in infinite dimensions. The control objective is to find,

E-mail address: vio@utgjiu.ro.

http://dx.doi.org/10.1016/j.jmaa.2014.03.052
0022-247X/

in a class of admissible control laws, the one which minimizes an infinite-horizon linear quadratic cost criterion under stabilizability and detectability conditions. As usually, the design of the optimal control is related to the global solvability of an associated Riccati-type differential equation (RDE) defined on a certain infinite-dimensional ordered Banach space. To avoid the complicated form of the coefficients of this particular RDE, we choose to study the solution properties for a class *of generalized RDEs* (GRDEs) which includes as special cases most of the known Riccati equations of control. In the general case the proofs are rather the same as in the case of RDEs, the notation is more convenient and the obtained results can be applied to other control problems. Such GRDEs were studied in [26] by using linear matrix inequality (LMI) techniques (see also [5] for the finite dimensional case). Assuming stabilizability hypotheses, [26] provides necessary and sufficient conditions for the existence of certain global solutions such as maximal, minimal and stabilizing solutions. Unlike [26], in this paper we investigate the existence of bounded and stabilizing solutions for GRDEs, under detectability conditions (see Theorem 9). The proofs are nonstochastic and based on operator theory. A key role in this operatorial approach is played by the asymptotic behavior of some positive evolution operators with Lyapunov type generators, defined on ordered Banach spaces. The elements of these Banach spaces are infinite sequences of either linear and bounded operators, trace class operators or Hilbert–Schmidt operators. This situation is characteristic to the infinite-dimensional case and increases the difficulty of the proofs (see [8–10] and [5] for a comparison with the finite dimensional case).

In order to apply the results on GRDEs to the stochastic optimal control problem, we need to establish the operatorial equivalent for the stochastic notions of stability, stabilizability and detectability. To this end, we give a version of Ito's formula which applies to infinite-dimensional stochastic processes with infinite MJs and two representation formulas for the solutions of linear SDEs with infinite MJs and MN. These results extend the ones obtained in [4,6] for Markov processes with finite state space and finite dimensional SDEs.

The paper is organized as it follows. Sections 2 and 3 present the notation, some preliminary results and the statement of the optimal control problem. In Section 4 we introduce a notion of detectability for pairs of operator valuated functions which extends the one in [5] and the stochastic detectability notion. In the discrete-time framework a similar notion was considered in [27]. Then we show that a classical result from the theory of Riccati equations remains true for a more general class of nonlinear differential equations (so called GRDEs) defined on ordered Banach spaces. This is the main result of this section and proves that under detectability conditions, any global, bounded and nonnegative solution of GRDEs is stabilizing. In Section 5 we give a generalized Ito's formula and a representation result for the solutions of linear SDEs with infinite MJs and MN. On the basis of these results we then obtain deterministic characterizations of the stochastic stability, stabilizability and detectability properties that we need in the rest of the paper. In Section 6 we solve an optimal control problem for linear SDEs with infinite MJs and MN, which consists in minimizing an infinite horizon quadratic cost functional over a class of admissible controls. In this paper the optimal control is obtained with the stabilizing solution of RDEs, under stabilizability and detectability hypotheses. The results of Sections 4 and 5 as well as the ones of [26] may be applied for solving some other optimal control problems. We mention here the infinite-dimensional versions of the optimal control problems formulated in Chapter 5 of [6].

2. Notations

Through this paper H, U, V are real separable Hilbert spaces and $\mathcal{E}$ is a real Banach space.

2.1. Linear and bounded operators on Hilbert spaces

By $L(H, U)$ we denote the real Banach space of linear and bounded operators from H into U. If $H = U$ we use the short notation $L(H)$ instead of $L(H, H)$. A similar notation will be used if the Hilbert spaces H