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Sweep method for solving the Roesser type equation describing the motion in the pipeline

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ABSTRACT

The initial problem for the system of hyperbolic equations describing the motion in oil production with gas lift method is considered. Introducing a new variable which is the difference of gas pressure and volume (or gas–liquid mixture (GLM)) multiplied by a constant number (balancing unit of measurements), the original system of equations is reduced to the such form of equations which after appropriate discretization becomes a Roesser type discrete equation. Searching the new variable as a linear function of the volume of gas (or GLM), it is shown that the coefficients satisfy the two difference equations of the first order, one of which corresponds to the quadratic equation and the second is a linear difference equation of the first order whose coefficients depend on the solution of the first one. In the case when the volume of the assessment gas and the motion (initial conditions) are constant at the mouth, it is shown that the results obtained by the Roesser model coincide with the known results, where the concrete analytical expression for the parameter of the balancing unit of measurements is provided.

1. Introduction

As is known, in different stages of oil production there exist numerous methods one of which is the gas-lift method [4–6,18,20]. There exist different ways for modeling [4,12,18] and then solving the corresponding equations arising in the gas-lift [3,8], where at first partial differential equations at time or at coordinates, are averaged [5,9]. Then the solution of corresponding ordinary differential equations is studied. The identification problems [17] of definition of the hydraulic resistance coefficient [6], formation of GLM [4–6] are investigated. Note that these problems concerning averaging are approximate and the obtained results are not in general adequate for the initial problem. Therefore, it is natural to study the initial problem where the motion is already described by the first order hyperbolic equation. On the other hand, in the gas-lift method, in the ring space and lift the motions are described by partial differential equations, but on the layer by the finite-difference equation. Therefore, it makes sense to discretize these partial equations and then to describe the general system of partial equations for the gas-lift process by a finite-difference equations.

It is shown that the general systems of equations can be reduced [11] to the useful form by using the Roesser model [2,13,19]. For these Roesser difference equations [2] the sweep method is suggested that makes easy the finding of approxi-

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mate solutions of initial hyperbolic equations coinciding with the known solutions by the more degree of accuracy. Note that for the periodic solutions of Roesser equation can be used the results [7], which can increase the debit in oil production.

2. Problem statement. Roesser type equation in continuous case

The Cauchy problem for a system of the first order hyperbolic type differential equations is considered [10,14]

$$\begin{cases} \dfrac{\partial P(x,t)}{\partial x} = -\dfrac{c}{F}\cdot\dfrac{\partial Q(x,t)}{\partial t}, \\ \dfrac{\partial Q(x,t)}{\partial x} = -F\dfrac{\partial P(x,t)}{\partial t} - 2aQ(x,t), \quad x\in R, \ \ t>0, \end{cases} \tag{1}$$

$$\begin{aligned} &P(x,0) = P_0(x), \\ &Q(x,0) = Q_0(x), \ \ x\in R. \end{aligned} \tag{2}$$

where c, F, Q are constants and defined as in [4,5,7,12,18,20], $P_0(x)$ and $Q_0(x)$ known continuous functions in $x\in R$.

Note that immediately after discretization the problems (1) and (2) it is impossible to obtain the Rosser model [19] because the derivatives $\frac{\partial P}{\partial x}$ and $\frac{\partial Q}{\partial t}$, also $\frac{\partial Q}{\partial x}$ and $\frac{\partial P}{\partial t}$ are involved into the same equation of the considering system. Therefore for applicability of the Roesser scheme [2] to Eq. (1), we use the following substitution into (2)

$$P(x,t) = R(x,t) + \alpha\, Q(x,t). \tag{3}$$

In (3) α is a parameter that balances the unit of measurements between P and Q.

Then we can write system (1) in the form:

$$\begin{cases} \dfrac{\partial R(x,t)}{\partial x} = -\alpha\dfrac{\partial Q(x,t)}{\partial x} - \dfrac{c}{F}\cdot\dfrac{\partial Q(x,t)}{\partial t}, \\ \dfrac{\partial Q(x,t)}{\partial x} = -F\left[\dfrac{\partial R(x,t)}{\partial t} + \alpha\dfrac{\partial Q(x,t)}{\partial t}\right] - 2aQ(x,t), \end{cases}$$

or

$$\begin{cases} \dfrac{\partial R(x,t)}{\partial x} + \alpha\dfrac{\partial Q(x,t)}{\partial x} = -\dfrac{c}{F}\cdot\dfrac{\partial Q(x,t)}{\partial t}, \\ \dfrac{\partial Q(x,t)}{\partial t} = -\dfrac{1}{F\alpha}\dfrac{\partial Q(x,t)}{\partial x} - \dfrac{1}{\alpha}\dfrac{\partial R(x,t)}{\partial t} - \dfrac{2a}{F\alpha}Q(x,t). \end{cases} \tag{4}$$

Now, taking into account the second equation of system (4) in the first one, we have

$$\begin{cases} \dfrac{\partial R(x,t)}{\partial x} = -\alpha\dfrac{\partial Q(x,t)}{\partial x} - \dfrac{c}{F}\left[-\dfrac{1}{F\alpha}\dfrac{\partial Q(x,t)}{\partial x} - \dfrac{1}{\alpha}\dfrac{\partial R(x,t)}{\partial t} - \dfrac{2a}{F\alpha}Q(x,t)\right], \\ \dfrac{\partial Q(x,t)}{\partial t} = -\dfrac{1}{F\alpha}\dfrac{\partial Q(x,t)}{\partial x} - \dfrac{1}{\alpha}\dfrac{\partial R(x,t)}{\partial t} - \dfrac{2a}{F\alpha}Q(x,t), \end{cases}$$

After grouping we obtain

$$\begin{cases} \dfrac{\partial R(x,t)}{\partial x} = \left(\dfrac{c}{F^2\alpha} - \alpha\right)\dfrac{\partial Q(x,t)}{\partial x} + \dfrac{c}{F\alpha}\dfrac{\partial R(x,t)}{\partial t} + \dfrac{2ac}{F^2\alpha}Q(x,t), \\ \dfrac{\partial Q(x,t)}{\partial t} = -\dfrac{1}{F\alpha}\dfrac{\partial Q(x,t)}{\partial x} - \dfrac{1}{\alpha}\dfrac{\partial R(x,t)}{\partial t} - \dfrac{2a}{F\alpha}Q(x,t). \end{cases} \tag{5}$$

Let accept the notations

$$\begin{cases} \dfrac{\partial Q(x,t)}{\partial x} = W(x,t), \\ \dfrac{\partial R(x,t)}{\partial t} = \chi(x,t), \ \ x\in R, \ \ t>0. \end{cases} \tag{6}$$

From systems (5) and (6) we have:

$$\begin{cases} \dfrac{\partial R(x,t)}{\partial x} = \left(\dfrac{c}{F^2\alpha} - \alpha\right)W(x,t) + \dfrac{c}{F\alpha}\chi(x,t) + \dfrac{2ac}{F^3\alpha}Q(x,t) \\ \dfrac{\partial Q(x,t)}{\partial t} = -\dfrac{1}{F\alpha}W(x,t) - \dfrac{1}{\alpha}\chi(x,t) - \dfrac{2a}{F\alpha}Q(x,t), \ \ x\in R, \ \ t>0. \end{cases} \tag{7}$$

Thus, we obtain two Roesser type systems, (6) and (7).

Lemma. Let the data c, F, Q, $P_0(x)$, $Q_0(x)$ be from an initial problem (1) and (2) such that its solution exists and is unique. Then using transformation (3) the systems of Eq. (1) may be reduced to two systems (6) and (7), which are continuous analog of Rosser model.

Note. As one can see from the systems (6) and (7) using the immediate discretization there is the discrete Rosser model.