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$\mathcal{H}_\infty$ control for singular Markovian jump systems with incomplete knowledge of transition probabilities

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ABSTRACT

This paper proposes a $\mathcal{H}_\infty$ state-feedback control for singular Markovian jump systems with incomplete knowledge of transition probabilities. Different from the previous results where the transition rates are completely known or the bounds of the unknown transition rates are given, a more general situation where the transition rates are partly unknown and the bounds of the unknown transition rates are also unknown is considered. Moreover, in contrast to the singular Markovian jump systems studied recently, the proposed method does not require any tuning parameters that arise when handling non-convex terms related to the mode-dependent Lyapunov matrices and the corresponding self-mode transition rates. Also, this paper uses all possible slack variables related to the transition rates into the relaxation process which contributes to reduce the conservatism. Finally, two numerical examples are provided to demonstrate the performance of $\mathcal{H}_\infty$ mode-dependent control.

1. Introduction

Over the past few decades, Markovian jump systems (MJSs) have drawn a lot of attention due to the fact that MJSs are quite appropriate to represent practical system subject to sudden variation, random component failures, and abrupt environment changes in their structures. Thus, many problems about MJSs with mode transition applied by a Markov stochastic process have been solved [1–12]. Besides, MJSs have been widely applied in many practical systems such as manufacturing systems [13], networked systems [14], and electric power systems [15]. Recently, singular MJSs (SMJSs) have attracted a lot of consideration of researchers due to the fact that SMJSs can better describe the behavior of some physical systems than standard MJSs [16–21].

More recently, MJSs and SMJSs with uncertain or partly unknown transition probabilities have been progressed as a topic of great interest, since it is difficult to obtain accurate values of the transition probabilities [22–33]]. The author of [26] proposed less conservative stabilization conditions for MJSs by including slack variables related to transition rates. For nonlinear MJSs with incomplete transition description, an improved fuzzy control design method [23] and a separated approach [29] were introduced by making the full use of the bounds of the unknown transition rates. The author of [25] addressed stability and stabilization of continuous-time SMJSs with partly unknown transition probabilities. The problem of state feedback controller design for singular positive MJSs with partly unknown transition rates was studied [28]. In [30], finite-time $\mathcal{H}_\infty$ control for SMJSs with partly unknown transition probabilities was studied. However, some of the available

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properties about the transition rates cannot be incorporated into their result and also, to the best of our knowledge, $\mathcal{H}_\infty$ control problem for SMJSs with incomplete knowledge of transition probabilities has not been fully investigated, which is motivation behind this study.

This paper proposes the $\mathcal{H}_\infty$ state-feedback control for singular Markovian jump systems with incomplete knowledge of transition probabilities. The distinguishing characteristic of the proposed method is that it has an effect the need of less conservative stabilization conditions and $\mathcal{H}_\infty$ state-feedback control as an alternative to the foregoing results. Also, different from the previous results, a more general situation where the transition rates are partly unknown and the bounds of the unknown transition rates are also unknown is considered. Moreover, in contrast to the SMJSs studied recently, the proposed method does not require any tuning parameters that arise when handling non-convex terms related to the mode-dependent Lyapunov matrices P_i and the corresponding self-mode transition rates π_{ii}. Also, this paper uses all possible slack variables related to the transition rates into the relaxation process which contributes to reduce the conservatism. The resulting conditions are converted into the second-order matrix polynomials of the unknown transition rates and the LMI conditions are obtained from the second-order polynomials. Finally, two numerical examples are provided to demonstrate the performance of $\mathcal{H}_\infty$ mode-dependent control.

The notations used in this paper are fairly standard. For $x \in \mathbb{R}^n$, x^T means the transpose of x. For symmetric matrices X and Y, $X \geq Y$ or $X > Y$ denotes that $X - Y$ is positive semi-definite or positive definite, respectively. I is an identity matrix with appropriate dimensions. For any square matrix Q, $\mathbf{He}(Q) = Q + Q^T$. $\|x\|$ refers to Euclidean norm of the vector x, $\mathbf{E}[\,\cdot\,]$ denotes the mathematical expectation. The Lebesgue space $\mathcal{L}_2^+ = \mathcal{L}_2[0, \infty)$ consists of square-integrable functions over $[0, \infty)$. For any matrices A_i, A_{ij} and set $\mathcal{S} = \{s_1, s_2, \ldots, s_k\}$, $[A_i]_{i\in\mathcal{S}} \triangleq [A_{s_1},\ A_{s_2}, \ldots,\ A_{s_k}]$, and

$$[A_{ij}]_{i,j\in\mathcal{S}} \triangleq \begin{bmatrix} A_{s_1s_1} & A_{s_1s_2} & \cdots & A_{s_1s_k} \\ A_{s_2s_1} & A_{s_2s_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ A_{s_ks_1} & \cdots & \cdots & A_{s_ks_k} \end{bmatrix}.$$

Also, in symmetric block matrices, $(*)$ is used as an ellipsis for terms that are induced by symmetry.

2. Problem statement

Consider a singular Markovian jump system defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$\begin{aligned} E\dot{x}(t) &= A(r(t))x(t) + B(r(t))u(t) + B_w(r(t))w(t), \\ z(t) &= C(r(t))x(t) + D(r(t))u(t) + D_w(r(t))w(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^q$ is the disturbance which belongs to $\mathcal{L}_2^+$, $z(t) \in \mathbb{R}^p$ is the performance output, the matrix $E \in \mathbb{R}^{n\times n}$ is supposed to be singular with $\text{rank}(E) = r < n$, and $\{r(t),\ t > 0\}$ is a continuous-time Markov process on the probability space that takes the values in a finite set $\mathbb{N}_N^+ \triangleq \{1, 2, \ldots, N\}$ and has the mode transition probabilities:

$$Pr(r(t+\delta t) = j | r(t) = i) = \begin{cases} \pi_{ij}\delta t + o(\delta t), & if\ j \neq i, \\ 1 + \pi_{ii}\delta t + o(\delta t), & otherwise, \end{cases} \tag{2}$$

where $\delta t > 0$, $\lim_{\delta t\to 0} o(\delta t)/\delta t = 0$, and $\pi_{ij} \geq 0$, for $j \neq i$, is the transition rate from mode i at time t to mode j at time $t + \delta t$, which satisfies the following properties

$$0 = \sum_{j=1}^{N} \pi_{ij}, \tag{3}$$

$$0 \leq \mu_{ij}\pi_{ij}, \quad \forall i, j \in \mathbb{N}_N^+ \tag{4}$$

where $\mu_{ij} = \{\begin{smallmatrix} 1, & if\ i\neq j \\ -1, & otherwise \end{smallmatrix}$. To simplify the notation, A_i, B_i, $B_{w,i}$, C_i, D_i and $D_{w,i}$ denote the $A(r(t) = i)$, $B(r(t) = i)$, $B_w(r(t) = i)$, $C(r(t) = i)$, $D(r(t) = i)$ and $D_w(r(t) = i)$, respectively. To facilitate the later discussion, we also define the two additional sets with respect to the transition rates π_{ij} for $i, j \in \mathbb{N}_N^+$:

$$\mathbb{D}_K^i \triangleq \{j | \pi_{ij} \text{ is known for } i\},$$

$$\mathbb{D}_{UK}^i \triangleq \{j | \pi_{ij} \text{ is unknown for } i\}.$$

Remark 1. For all $i, j \in \mathbb{N}_N^+$, $j \neq i$, it is obvious that $\pi_{ij} \leq -\pi_{ii}$ and $\pi_{ii} \leq -\pi_i^+$ where $\pi_i^+ = \sum_{j\in\mathbb{D}_K^i} \pi_{ij}$. Besides, since the transition probability (2) is bounded on [0, 1], the value of π_{ii} cannot be smaller than $-1/\delta t$. Therefore, the transition rates π_{ij} satisfy the following additional properties:

$$0 \leq -\pi_{ij}(\pi_{ij} + \pi_{ii}), \quad \forall i, j \in \mathbb{N}_N^+, i \neq j, \tag{5}$$