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The extended generalized Störmer–Cowell methods for second-order delay boundary value problems☆

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ABSTRACT

This paper deals with the numerical solutions of second-order delay boundary value problems (DBVPs). The generalized Störmer–Cowell methods (GSCMs) for second-order initial value problems, proposed by Aceto et al. (2012), are extended to solve the second-order DBVPs. The existence and uniqueness criterion of the methods is derived. It is proved under the suitable conditions that an extended GSCM is stable, and convergent of order p whenever this method has the consistent order p. The numerical examples illustrate efficiency and accuracy of the methods. Moreover, a comparison between the extended GSCMs and the boundary value methods of first-order BVPs is given. The numerical result shows that the extended GSCMs are comparable.

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1. Introduction

In the past decades, delay boundary problems (DBVPs) have played the important role in modeling the real issues, such as heat conduct, impulsive phenomena, traveling wave, economic control and so forth (see e.g. [2,3]). Usually, it is difficult to get an exact solution of such a problem as the presentation of the delay item in DBVPs. Hence, one turns to consider using numerical methods to solve DBVPs. An effective approach to obtain the numerical methods of DBVPs is adapting the underlying methods of initial value problems (IVPs) or boundary value problems (BVPs).

As we know, the research on numerical methods for DBVPs is very few, although the investigation on numerical methods for delay initial value problems (DIVPs) have been presented in many references, seeing e.g. Bellen and Zennaro's monograph [4] and the references therein. Here, we give a brief review for the existed methods of DBVPs. For second-order DBVPs with time-dependent delay, Nevers and Schmitt [5] suggested a shooting method, Reddien and Travis [6] and Reddien [7] constructed a projection method and a midpoint method, Chocholaty and Slahor [8] considered a direct iterative method, Agarwal and Chow [9] proposed a finite difference method, Qu and Agarwal [10] gave a subdivision method, and Bellen and Zennaro [11] dealt with piecewise polynomial approximation. For second-order DBVPs with state-dependent delay, by combining second-order centered difference, Lagrange interpolation and Richardson extrapolation, Bakke and Jackiewicz [12] obtained the convergent schemes of second-order and third-order . More generally, for a class of functional BVPs, Cryer [13] derived a finite difference method and proved existence and convergence of the approximate solution.

It is well-known that the linear multistep methods (LMMs) of IVPs can be applied to solve BVPs (see e.g. [14,15]) if some appropriate initial and final values are supplied. Moreover, when a LMM is used as a boundary value method (BVM),

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it becomes possible that the so-called Dahlquist order barrier (cf. [16,17]) could be overcomed. As a typical example, by adapting the Störmer–Cowell methods of second-order IVPs, Aceto, Ghelardoni and Magherini [1] constructed a family of P_ν-stable BVMs which can arrive at arbitrary high order. Such methods are called *generalized Störmer–Cowell methods (GSCMs)*. In view of the advantages of GSCMs for IVPs, in the present paper, we will extend GSCMs to solve the following second-order DBVPs with delay $\tau > 0$:

$$\begin{cases} y''(t) = f(t, y(t), y(t-\tau)), & t \in [a, b], \\ y(t) = \varphi(t), \quad t \in [a-\tau, a], & y(b) = y_b, \end{cases} \tag{1.1}$$

where functions $\varphi(t) : [a-\tau, a] \to \mathbb{R}$ and $f : [a, b] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ are assumed to be smooth enough on their respective domains, and $f(t, y, z)$ satisfies the Lipschitz condition with constant $L > 0$ for all $(t, y_1, z_1), (t, y_2, z_2) \in [a, b] \times \mathbb{R} \times \mathbb{R}$:

$$|f(t, y_1, z_1) - f(t, y_2, z_2)| \le L(|y_1 - y_2| + |z_1 - z_2|). \tag{1.2}$$

The paper is organized as follows. In Section 2, for convenience of the subsequent analysis, we give a brief review to the underlying LMMs for second-order initial and boundary problems. In Section 3, we extend GSCMs to solve second-order DBVPs (1.1) and study the existence and uniqueness of the solution of the extended method. In Section 4, we derive a convergence result of the methods. In Section 5, we analyze the stability of the methods and give a numerical stability criterion. In Section 6, with some numerical examples, we further illustrate the efficiency and accuracy of the methods. In Section 7, we give a summary to the paper, and compare the extended GSCMs and the BVMs of first-order BVPs. The numerical result shows that the extended GSCMs are comparable.

2. A review to the underlying LMMs for second-order initial and boundary problems

In order to lay a basis for our investigation, in this section, we give a brief review on the underlying LMMs for second-order initial and boundary problems.

For second-order IVPs

$$y'' = f(t, y(t)), \quad t \in [a, b], \quad y(a) = y_0, \quad y'(a) = y_0', \tag{2.1}$$

the k-step LMMs with real coefficients α_j and β_j are given by (cf. [15])

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h^2 \sum_{j=0}^{k} \beta_j f_{n+j}, \quad n \ge 0, \tag{2.2}$$

where $t_n = a + nh$, $h = \frac{b-a}{N}$, $y_n \approx y(t_n)$, $f_n \equiv f(t_n, y_n) \approx f(t_n, y(t_n))$. Method (2.2) is said to be *consistent of order p* if

$$C_0 = C_1 = \cdots = C_{p+1} = 0 \quad \text{and} \quad C_{p+2} \neq 0 \tag{2.3}$$

with

$$\begin{cases} C_0 = \sum\limits_{i=0}^{k} \alpha_i, \quad C_1 = \sum\limits_{i=1}^{k} i\alpha_i, \quad C_2 = \frac{1}{2}\sum\limits_{i=1}^{k} i^2\alpha_i - \sum\limits_{i=0}^{k} \beta_i, \\ C_q = \frac{1}{q!}\sum\limits_{i=1}^{k} i^q\alpha_i - \frac{1}{(q-2)!}\sum\limits_{i=1}^{k} i^{q-2}\beta_i, \quad q = 3, 4, \ldots. \end{cases} \tag{2.4}$$

As stated in Henrici [15], LMMs (2.2) can be adapted into BVMs, where k_1 initial values and k_2 final values are requested and $k_1 + k_2 = k$. This type of adapted methods will be called *BVMs with* (k_1, k_2)*-boundary conditions*. As an example, Aceto et al. [1] derived the following 2ν-step 2ν-order *generalized Störmer–Cowell methods* (GSCMs) with (ν, ν)-boundary conditions:

$$y_{n+1} - 2y_n + y_{n-1} = h^2 \sum_{j=-\nu}^{\nu} \beta_{j+\nu} f_{n+j}, \quad n = \nu, \nu+1, \ldots, N-\nu, \tag{2.5}$$

where $h = \frac{b-a}{N}$, $\beta_j = \beta_{2\nu-j}$, $j = 0, 1, \ldots, 2\nu$. To apply this type of methods to solve the second-order BVPs

$$y''(x) = f(x, y), \quad x \in [a, b], \quad y(a) = y_a, \quad y(b) = y_b,$$

besides the known values $y_0 (= y_a)$ and $y_N (= y_b)$, we still need to give the $\nu - 1$ initial values $y_1, y_2, \ldots, y_{\nu-1}$ and $\nu - 1$ final values $y_{N-\nu+1}, \ldots, y_{N-1}$. These extra values can be obtained by the following $(2\nu - 2)$ auxiliary schemes:

$$y_{i-1} - 2y_i + y_{i+1} = h^2 \sum_{j=0}^{2\nu-1} \beta_j^{(i)} f_j, \quad i = 1, 2, \ldots, \nu-1, \tag{2.6}$$

and

$$y_{k-1} - 2y_k + y_{k+1} = h^2 \sum_{j=0}^{2\nu-1} \beta_j^{(i)} f_{k-i+j+1}, \qquad i = \nu+1, \nu+2, \ldots, 2\nu-1; \; k = N+i-2\nu. \tag{2.7}$$

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