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Synchronization for coupled nonlinear systems with disturbances in input and measured output

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ABSTRACT

This paper considers the synchronization for coupled nonlinear systems with disturbances in input and measured output. By defining a controlled output, the synchronization problem is converted to a special suboptimal $\mathcal{H}_\infty$ control problem. Precisely speaking, for a given disturbance attenuation level, we need to design a distributed output-feedback protocol such that the closed-loop system asymptotically reaches output synchronization when there do not exist disturbances, and the L_2-gain from disturbances to the controlled output is less than the given level. We first consider the case that each agent is incrementally passive. Secondly, we consider the case that each agent is feedback incrementally passive and the measured output is not influenced by disturbances. Finally, two numerical examples are presented to illustrate the effectiveness of the proposed control law.

1. Introduction

Synchronization (consensus) of a network is a common phenomenon occurring in many fields of science, e.g., physics, biology, society, and engineering. The research on synchronization problem in a network of systems has witnessed considerable efforts in the last decade, such as consensus of multi-agent systems [1–8], synchronization of complex networks [9–18]. In reality, nodes (agents) in a network are usually influenced by external disturbances and subject to communication noises. Hence, it is reasonable to ask how to achieve some prescribed performance relative to perfect synchronization when the network subjects to disturbances.

When the network is influenced by disturbances and communication delays $\mathcal{H}_\infty$ performance is often expected to achieve, such as $\mathcal{H}_\infty$ consensus of multi-agent systems [4–7,19], the stability of networked control system with time-varying delays [19]. In Li et al. [4], the authors investigated the $\mathcal{H}_\infty$ performance region of multi-agent systems, which was used to investigate distributed $\mathcal{H}_\infty$ consensus [5]. Liu and Jia [6] proposed a novel dynamic output feedback and achieved $\mathcal{H}_\infty$ consensus of multi-agent systems with switching topology. For high-order multi-agent systems, [7] studied the $\mathcal{H}_\infty$ consensus problem. Each agent in the above multi-agent systems is described as an integrator or a linear time-invariant system. However, to better understand the collective behavior of real network, each agent should be described as a nonlinear system. Hence, it is necessary to study the synchronization of a network of nonlinear systems subject to external disturbances.

Generally speaking, for a network of systems with external disturbances and uncertainty [20–23], the network is hard to achieve synchronization. Specially, synchronization phenomenon can be achieved for some specific disturbances, e.g.,

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constants, sinusoids, etc [24,25]. By using internal model control approach, [24] studied output agreement in networks of heterogeneous nonlinear dynamical systems affected by some specific disturbances. In Bai and Shafi [25], the authors considered output synchronization of a network of nonlinear systems that satisfy an additional incremental passivity property. When the agents are influenced by general disturbances, it is expected to achieve $\mathcal{H}_\infty$ synchronization (see problem statement). Recently, it was shown that the $\mathcal{H}_\infty$ synchronization is effective for disturbance attenuation in a network of a class of nonlinear systems [20].

Motivated by the above observations, this paper studies the synchronization of a network of nonlinear systems, where each agent is subject to admissible disturbances in input and measured output. In order to measure the output disagreements among agents, we define a new output to convert the problem to a special suboptimal $\mathcal{H}_\infty$ control problem. If each agent is incrementally passive, then we design a distributed output-feedback protocol to solve the converted problem. The results were partly reported in our paper [26]. If each agent satisfies feedback incremental passivity property and the measured output is not influenced by disturbance, then we design a distributed output-feedback protocol to solve the problem. Finally, two numerical examples are presented to illustrate the effectiveness of the obtained results.

The paper is organized as follows. Section 2 presents some preliminaries and problem statement. Section 3 investigates $\mathcal{H}_\infty$ synchronization problem for agents subject to external disturbances in input and measured output. Section 4 gives two numerical examples to illustrate the obtained results. Section 5 summarizes our conclusions and describes future work.

The following notations are used throughout this article. The notation $\mathcal{L}_2^m[0, T)$ denotes the set of all vector-valued functions $z : [0, T) \to \mathbb{R}^m$ which satisfy $\int_0^T ||z(t)||^2 dt < \infty$, where $||z||^2$ denote the square norm of vector z. The symbol I_N denotes the N-dimensional identity matrix. Let $\mathbf{0}_N(\mathbf{1}_N)$ be the N-dimensional column vector with each entry being 0(1) . In symmetric block matrices, we use $*$ as an ellipsis for the terms that are introduced by symmetry. The notation $P > 0(< 0)$ means P is real symmetric and positive definite (negative definite). The notation $\otimes$ denotes the Kronecker product. A function $g : \mathbb{R} \to \mathbb{R}$ is said to be continuously differentiable or $\mathcal{C}^1$ provided g and $\dot{g}(\cdot)$ are continuous functions.

2. Preliminaries and problem statement

2.1. Preliminaries

In this subsection, we present some basic notations and lemmas about graph. An undirected graph is defined by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of all nodes, and an edge $(i, j) \in \mathcal{E}$ if and only if nodes i and j connect. Let $A = (a_{ij}) \in \mathbb{R}^{N\times N}$ be adjacency matrix of the graph $\mathcal{G}$, where $a_{ij} = 1, i \neq j, a_{ii} = 0$. The corresponding Laplacian matrix is $L = (l_{ij}) \in \mathbb{R}^{N\times N}$. $l_{ii} = -\sum_{j=1, j\neq i} l_{ij}, l_{ij} = -a_{ij}, i \neq j$. A path of $\mathcal{G}$ from i_1 to i_{j+1} is a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \ldots, (i_j, i_{j+1})$. The graph $\mathcal{G}$ is said to be connected if there exists a path between any two distinct nodes.

Lemma 1 [27]**.** *Let $L \in \mathbb{R}^{N\times N}$ be the Laplacian matrix of an undirected graph $\mathcal{G}$. Then $\frac{1}{\sqrt{N}}[1 \ \ldots \ 1]^{\mathrm{T}}$ is the eigenvector of Laplacian matrix L associated with eigenvalue $\lambda_1 = 0$. Moreover, if the graph $\mathcal{G}$ is connected, then the other $N - 1$ eigenvalues $\lambda_2, \ldots, \lambda_N$ are positive constants.*

Lemma 2 [28]**.** *Let $L_c \in \mathbb{R}^{N\times N}$ be a projection matrix with*

$$L_c = I_N - \frac{1}{N}\mathbf{1}_N\mathbf{1}_N^{\mathrm{T}}.$$

Then the following statements hold:

(1) *The eigenvalues of L_c are 1 with multiplicity $N - 1$ and 0 with multiplicity 1. The vectors $\mathbf{1}_N^{\mathrm{T}}$ and $\mathbf{1}_N$ are the left and right eigenvectors of L_c associated with the zero eigenvalue, respectively.*
(2) *There exists an orthogonal matrix $U \in \mathbb{R}^{N\times N}$ such that*

$$U^{\mathrm{T}} L_c U = \begin{bmatrix} I_{N-1} & \mathbf{0}_{N-1} \\ * & 0 \end{bmatrix},$$

and the last column of U is $\mathbf{1}_N/\sqrt{N}$. Moreover, let $L \in \mathbb{R}^{N\times N}$ be the Laplacian of an undirected graph, then

$$U^{\mathrm{T}} L U = \begin{bmatrix} L_1 & \mathbf{0}_{N-1} \\ * & 0 \end{bmatrix},$$

where $L_1 \in \mathbb{R}^{(N-1)\times(N-1)}$ is positive definite if and only if the graph is connected.

Remark 1. Suppose that $U = [U_1 \ \mathbf{1}_N/\sqrt{N}]$. Then we can easily obtain $UU^{\mathrm{T}} = U_1 U_1^{\mathrm{T}} = L_c$. Note that $\frac{1}{\sqrt{N}}[1\ 1 \ldots \ 1]^{\mathrm{T}}$ is the eigenvector of Laplacian matrix L associated with eigenvalue $\lambda_1 = 0$. Moreover, if the graph $\mathcal{G}$ is connected, then the other $N - 1$ eigenvalues $\lambda_2, \ldots, \lambda_N$ are positive constants, that is, the eigenvalues of L_1 are positive [27].