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Evolutionary snowdrift game with rational selection based on radical evaluation

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ABSTRACT

Considering some phenomena observed in the real world, we introduce a rational selecting mechanism based on radical evaluation into evolutionary snowdrift game. In the proposed model, players are endowed with a sense of rationality, which helps evaluate behaviors radically and select neighbors with different attractiveness. Those neighbors who made a preferable strategy and got more payoffs compared with the anti-strategy will have more attractiveness as references. It is found that the selection based on radical evaluation significantly enhances the level of cooperation on regular networks with large neighborhood size K and scale-free networks over a wide range of cost-to-benefit ratio r. Discussions for the transition of spatial patterns and strategy degree distribution at some critical values of the payoff parameter show the effects of the proposed selecting mechanism. The findings may be helpful in understanding cooperative behavior in natural and social systems consisting of rational selection with radical evaluation.

1. Introduction

Cooperation widely exists in various fields from complex biological to economic and social networks [1–7]. Evolutionary game theory provides a common effective framework for researching how to describe and understand the emergence of cooperative behavior among selfish individuals in a competing setting [8–18].

Among all game models within this framework, two simple classic kinds of games, the prisoner dilemma game (PDG) [19–21] and the snowdrift game (SG) [22,23], attract the most social researches and discussions. The original PDG and SG are two-person game processes, during which each single agent decides to cooperate (i.e., C strategy) or defect (i.e., D strategy) simultaneously. Two competing agents will receive the reward (R) or punishment (P) respectively if they both cooperate or defect. Once they make different strategies, the defector can exploit the cooperator and obtain the highest payoff (T), while the cooperator will get the sucker's payoff (S). There exists some restriction on payoffs in PDG and SG respectively. In the PDG, these payoffs satisfy the following rankings $T > R > P > S$ and $2R > (T + S)$. The only difference in SG must be ordered as $T > R > S > P$ [24,25].

Though it seems that the difference between the two payoff orders is slightly unimportant, it turns out to be a pronounced impact on the cooperative behavior when individuals are facing such an interacting dilemma [22,26]. It comes to a classical conclusion in PDG that defection is always the best strategy for both competing individuals, while cooperators cannot survive. However, cooperative behavior may be encouraged in SG. The opposite strategy is always the best response

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to the competitor. In general, there is a strict rule in discussing the emergence of cooperative behavior owing to the difficulties in assessing proper payoffs, and SG is often regarded as a good and realistic alternative to PDG in describing competing situations [27].

Since Axelrod proposed the groundwork on repeated games, extensive investigations has been paid to the evolutionary dynamics of cooperative behavior in structured or connected populations [4]. Cooperation could be promoted in the evolutionary game theory by allowing the players to interact repeatedly [28]. On one hand, spatial structures of individual interaction are found to result in emergence and persistence of cooperation. For instance, Nowak and May's seminal works indicate that spatial structure on regular lattice is capable to facilitate the fraction of cooperation [29]. While Hauert and Doebeli examined the snowdrift game on square lattices and found out that spatial structure often inhibits the evolution of cooperation, in sharp contrast to the result of PDG [22]. Besides these previous researches, relevant works have shown that the evolution of cooperation can be influenced apparently in other complex topologies, including small-world and scale-free properties [30–33]. On the other hand, the coevolution of network topology and mechanisms of game dynamics are proposed to encourage between interacting individuals. Nowak [34] discussed five mechanisms for the evolution of cooperation: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection. In addition, diverse mechanisms have been discussed in various conditional situations, including individual migration or mobility [35,36], reward or punishment [37,38], individual aspiration [39,40], *group diversity* [41–43], payoff weighting [44,45], and etc.

Among the previous work, the effects of preferential selection among individuals have been researched in the study of evolutionary games on networks, in which different individuals may have different attractiveness and preferential selection in social systems. When updating their strategies, the individuals may not randomly choose a neighbor to refer to [44–47]. Motivated by these studies, we argue that individuals usually make decisions based on radical evaluation in snowdrift game. During the modified evolution game, each individual is endowed with a certain extent of rational sense, with which it can assess the behaviors and judgments of the neighbors radically. Those neighbors who made a preferable strategy and got more payoffs compared with the anti-strategy will have more attractiveness as references, otherwise they fail to achieve recognition. We examine whether it hinders the cooperative behavior and in particular, how this mechanism affects the evolution of cooperation on the regular networks as well as on the scale-free network for different levels of rational adoptions.

2. The model

The original SG model can be described by the following scenario: two drivers trapped on either side of a huge snowdrift. They can either choose to cooperate (C) by shoveling the snow or to remain in the car (D) in any one negotiation. If they both choose C, then they both gain benefit b of getting back home and share the labor cost c of shoveling, i.e., both get payoff $R = b - c/2$. If both remain in the car, they will still be trapped by the snowdrift and get punitive payoff $P = 0$. If one shovels C while the other one stays in the car for D, they both can get home, but the defector avoids the labor cost c and obtains the highest payoff $T = b$, whereas the cooperator receives the sucker's payoff $S = b - c$. For simplicity, we assign R=1 to characterize all payoffs by a single parameter $r = c/2 = c/(2b - c)$, defined as the cost-to-benefit ratio. The payoffs are then rewritten in a rescaled payoff matrix

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & 1-r \\ 1+r & 0 \end{pmatrix} \end{array}$$

where r generally lies between 0 and 1.

Here, we introduce the rules of the rational selection mechanism. Consider that each competing player is placed on the nodes of a certain network and gets a total payoff by playing only with connected neighbors simultaneously in every round. During the evolutionary process, there exist two parallel selection methods (Type A and Type B) for each player in every round. Only one method will be implemented for one's selection. In the Type A, implemented as a probability p, all the neighbors are considered different for their judgments, and players are allowed to select one of them with different attractiveness. The Type B is implemented as a probability 1 $-p$, with which a player choose one uniformly at random from all the neighbors. The probability of x selecting a neighbor y can be defined as

$$\Pi_y = \frac{E_y}{\sum_{z \in \Omega_x} E_z} \tag{1}$$

$$E_y = \begin{cases} \tilde{E}_y, & 0 < \xi_x \le p \\ \frac{1}{k_x}, & p < \xi_x < 1 \end{cases} \tag{2}$$

where Ω_x is the community composed of x's neighbors, E_y denotes the level of attractiveness of y, ξ_x is a random number uniformly distributed in the range (0,1) for x carrying out the selection, and k_x is the degree of x. Whichever the method is, the deterministic ingredient of the selection is the attractiveness kernel function E.

In Type B, all neighbors have the same attractiveness $1/k_x$ in terms of x. In Type A, The attractiveness $\tilde{E}$ is dependent on whether a player got more payoffs with the current strategy by means of self-questioning and radical evaluation. Such an