



A complete analysis of the global dynamics of a diffusive predator and toxic prey model



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ABSTRACT

Considering many species can release toxic substances to protect themselves against predators, a diffusive predator and toxic prey system with spatial heterogeneity in predator and prey populations has been investigated. For this system, we give a complete and rigorous analysis of the global dynamics with the boundedness, globally asymptotical stability, transcritical bifurcation, Hopf bifurcation and its direction, and the stability of the bifurcating periodic solutions. Meanwhile, we consider the effects of toxins produced by the prey on the dynamic behavior. The consequence of the global stability of the coexistence equilibrium is that the toxin's intrinsic characteristic will not change the stability of the system irreversibly. Our results show that the toxin-produced by the prey (phytoplankton) may be used as a bio-control agent for the Harmful Algal Bloom problems.

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1. Introduction

In ecological system, many species can release toxic substances to protect themselves against predators, such as rattlesnake, spider, jellyfish. Furthermore, scientists have known for a long time that many plants protect themselves from insect attacks by producing toxins which either ward off or kill insects. Recently, expert claim that people should be aware that certain algae can release toxins and those flourished can be harmful, especially in polluted waters which are full of carbon, nitrogen and phosphorus etc. Presence of residual sodium carbonate acts as catalyst for the algae to bloom by providing dissolved carbon dioxide for enhanced photosynthesis in the presence of nutrients. Hence, the Harmful Algal Blooms (HABs) is an algal bloom that causes negative impacts to other organisms via production of natural toxins or mechanical damage to other organisms, or by other means. A global increase in harmful plankton blooms has produced great socio-economic damage in the last two decades [1–3]. In homogeneous space, there has been considerable scientific attention towards HABs and its control [4–9]. However, spatial diffusion is ubiquitous and makes the spatiotemporal dynamic behaviors more complicated and rich. In a number of previous studies, Roy [10] explore the competitive effects of toxin-producing phytoplankton on phytoplankton and zooplankton species undergoing spatial movements in the subsurface water; Chakraborty et al. [11] demonstrate how an inhomogeneous biomass distribution of phytoplankton emerges over space due to the toxin effect; Rao [12] simply obtain the conditions for global stability and for diffusive instability of a positive equilibrium of system (1.1) in terms of numerical simulation. In this paper, we will give a complete and rigorous analysis of the global

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dynamics of a predator and toxic prey model in non-homogeneous space

$$\begin{cases} \frac{\partial u_1}{\partial t} = d_1 \frac{\partial^2 u_1}{\partial x^2} + ru_1 \left(1 - \frac{u_1}{k}\right) - \frac{\mu u_1 u_2}{\alpha + u_1}, & x \in (0, l\pi), t > 0, \\ \frac{\partial u_2}{\partial t} = d_2 \frac{\partial^2 u_2}{\partial x^2} + u_2 \left(\frac{(\mu_1 - \rho)u_1}{\alpha + u_1} - \delta\right), & x \in (0, l\pi), t > 0, \\ \partial_n u_1 = \partial_n u_2 = 0, & x = 0, l\pi, t > 0. \end{cases} \tag{1.1}$$

The following assumptions for model (1.1) are made:

- The variables $u_1(t, x)$ and $u_2(t, x)$ represent the densities of the prey and predator species at time t and at position x , respectively.

- The parameter $d_i (> 0) i = 1, 2$ is diffusion coefficient of population u_i . ∂_n is the directional derivative normal to $x = 0$ and $x = l\pi (l > 0)$. The homogeneous Neumann boundary condition indicates that there is zero population flux across the boundary.

- The constant μ is the maximum uptake rate, μ_1 denotes the ratio of biomass conversion, ρ_1 denotes the rate of toxic substances produced by per unit biomass, δ is the natural death rate, corresponding to the predator.

- In this context, the toxic substances produced by the prey have a direct effect on the mortality of zooplankton. Such extra mortality further reduces predator abundance and affects predation pressure on prey which is an indirect effect of toxin on the grazing pressure of the predator, and thus is one of the most important parameters. We assume $H(u_1) = \frac{\mu_1 u_1 u_2}{\alpha + u_1}$ represents the Holling type II functional response for the grazing of the prey by the predator and α is the half saturation constant. This is a reasonable assumption to describe the law of predation, because the prey is the main food source for the predator (since the number of predators is strongly dependent on the number of prey, and the prey extinction will almost surely lead to predator extinction). Furthermore, we assume that the prey's contribution to the predator's growth is proportional to the toxic substance's contribution to the predator's death. Hence, we use $T(u_1) = \frac{\rho u_1 u_2}{\alpha + u_1}$, which is proportional to $H(u_1)$, to describe the distribution of toxic substance which ultimately contributes to the death of prey populations. In this paper, we will be interested in the dynamics of the system with same functional form for $T(u_1)$ and $H(u_1)$.

- Here, we observe that if $\mu_1 - \rho - \delta < 0$ then the growth of the predator species is negative, i.e., $u_2(u_1(\mu_1 - \rho)/(\alpha + u_1) - \delta) < 0$. Throughout our analysis, we assume that

$$\mu_1 - \delta < \rho.$$

In this setting, we further give a complete and rigorous analysis of the global dynamics of (1.1), and obtain the boundedness, globally asymptotical stability, the transcritical bifurcation, Hopf bifurcation and its direction, and the stability of the bifurcating periodic solutions, and how the toxic parameter affects the dynamics of the ecosystem. For the diffusive model, our main technique of this work to accomplish our spectral and bifurcation analyses is the adjoint theory of partial differential equations (PDEs) in the neighborhood of the coexistence equilibrium state. The linearized equation around the equilibrium state is given by an operator that mixes the modes of the Laplacian, and the results for the reaction diffusion system are developed by the case without diffusion (ODEs).

It is well known that the global stability of a diffusive population model implies the existence and nonexistence of a nonconstant steady state. Recently, there are many results of the global dynamics of the diffusive predator–prey model with ratio-dependent functional response, such as

$$\begin{cases} u_{1t} = d_1 \Delta u_1 + ru_1 \left(1 - \frac{u_1}{k}\right) - u_2 p(u_1, u_2), & x \in \Omega, t > 0, \\ u_{2t} = d_2 \Delta u_2 + su_2 \left(1 - \frac{u_2}{q(u_1, u_2)}\right), & x \in \Omega, t > 0, \\ \partial_n u_1 = \partial_n u_2 = 0, & x = \partial \Omega, t > 0 \end{cases} \tag{1.2}$$

where p is an increasing function with respect to u_1 and decreasing function of u_2 , respectively; and q is increasing function of u_1 and u_2 . The functional response $p(u_1, u_2)$ represents various types:

Holling types I–III [13–16], $p = \mu u_1$, $p = \frac{\mu u_1}{\alpha + u_1}$, $p = \frac{\mu u_1^2}{(a + u_1)(b + u_1)}$,

Beddington–DeAngelis type [16,17], $p = \frac{\mu u_1}{\alpha + u_1 + m u_2}$,

more complicated types $p = \frac{\mu u_1^2}{u_1^2 + m u_2^2}$ [18] and $p = \mu \sqrt{u_1}$ [19].

In addition, the function q has many forms for example $q = \gamma u_1$, $q = \gamma u_1 + u_2$ and so on. In term of the above two functions with quasi monotone properties, we can obtain the globally asymptotical stability by using the iterative method and the standard comparison theorem coupled upper and lower solutions. However, it is difficult to obtain the result of global dynamics according to the method, since the term $\frac{(\mu_1 - \rho)u_1}{\alpha + u_1} - \delta$ in the second equation of (1.1) is independent on the variable u_2 . In this paper, we will establish the global results by constructing the Lyapunov function under certain range of toxic parameters. Meanwhile, the coexistence constant steady state of system (1.1) is unstable for any diffusions $d_1, d_2 > 0$ in the other region of toxin coefficients, which implies this is not Turing instability [20].

It is well known that there may be a unique limit cycle when the stable coexistence equilibrium becomes unstable for ODEs [21]. Furthermore, the periodic orbit of ODEs gets a spatially homogeneous periodic orbit for PDEs under Neumann

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