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Analytic solutions of multiple moving cracks in an orthotropic layer bonded to an orthotropic FGM coating

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A B S T R A C T

In this paper, the dynamic behavior of an orthotropic substrate weakened by moving cracks and reinforced by a non-homogenous coating is studied. First, the solution to the screw dislocation in an orthotropic strip with imperfect orthotropic functionally graded coating is obtained. Then, for the anti-plane analysis of cracks, the screw dislocations are distributed along the crack lines and the dislocation solution is used to derive integral equations for dislocation density functions on the surface of cracks. The effects of non-homogeneity parameters, geometrical parameters and bonding coefficient on the stress intensity factors are investigated.

1. Introduction

One of the important potential applications of functionally graded materials (FGMs) is in the area of thermal barrier coatings (TBCs) technology. Used as coating and interfacial zones, FGMs help to reduce mechanically and thermally induced stresses caused by the material property mismatch and to improve the bonding strength. Typical applications of FGMs include thermal barrier coatings of high temperature components in gas turbines, surface hardening for tribological protection, and as inter-layers in microelectronic and optoelectronic components. While the ceramic in an FGM supports thermal barrier effects and protects the metal from oxidation and correction, the metal increases the strength of the FGM. The study of FGMs reveals that the residual stress can be effectively relaxed by using an FGM coating in a coating–substrate composite.

In many engineering applications, non-homogeneous structures may be subjected to dynamic loadings. The dynamic manipulation of such structures may lead to crack formation and eventually the failures of the structures. During the last decade, the problem of dynamic crack propagation in a coating–substrate structures has been studied by many other researchers, both theoretically and experimentally. The knowledge of crack propagation in non-homogeneous orthotropic materials is important in designing components made of FGMs and improving their fracture toughness. The problems of crack propagation at constant speed can be classified into three categories depending on the boundary conditions [1]. The prototype problem of the first category is the two-dimensional Yoffe problem of a crack with fixed length propagating in a body.

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The effect of crack propagation velocity on the stress intensity factor has been studied by Sih [2]. The solution for this complex elastodynamic problem has been sought under various assumptions. Later, Sih and Chen [3] studied the dynamic behavior of a moving crack in layered composites while Danyluk and Singh [4] obtained closed form solutions for a finite length crack moving with constant velocity in an orthotropic layer. Wang and Meguid [5] introduced a theoretical and numerical treatment of a finite crack propagating in an interfacial layer with spatially varying elastic properties under the anti-plane loading. Wang and Williams [6] analyzed the dynamic crack propagation in tapered double cantilever beam specimens with the use of beam theory and the finite element method.

The dynamic crack propagation in FGMs under the plane elastic deformation using Fourier transform technique was investigated by Meguid et al. [7]. Jiang and Wang [8] studied the dynamic behavior of a Yoffe type crack propagating in a functionally graded interlayer bonded to dissimilar half planes. Chi and Chung [9] obtained the stress intensity factor for cracked multi-layered and functionally graded material coatings of a coating–substrate composite. The dynamic stress intensity factor and strain energy density for moving crack in an infinite strip of functionally graded material subjected to anti-plane shear was determined by Bi et al. [10]. A finite crack with constant length propagating in the functionally graded orthotropic strip under in plane loading was investigated by Ma et al. [11]. Therein, the effects of material properties, thickness of the functionally graded orthotropic strip and the speed of the crack propagating upon the dynamic fracture behavior were studied.

Das [12] investigated the interaction between three moving collinear Griffith cracks under anti-plane shear stress situated at the interface of an elastic layer overlying a different half plane. The problem of a Griffith crack of constant length propagating at a uniform speed in a non-homogeneous plane under uniform load was studied by Singh et al. [13]. Wang and Han [14], considered the problem of a moving crack in a non-homogeneous material strip. They found that the maximum anti-plane shear stress around the crack tip is a suitable failure criterion for moving cracks. In another paper, Wang and Han investigated the dynamic behavior of a crack moving at the interface between an FGM layer and homogeneous substrate [15]. The finite crack with constant length (Yoffe-type crack) propagating in a functionally graded strip with spatially varying elastic properties between two dissimilar homogeneous layers under in-plane loading was studied by Cheng and Zhong [16]. The dynamic delamination problems have been studied for steady-state crack growth in fiber reinforced composites by Gereco and Lonetti [17]. Bruno et al. [18] also investigated the dynamic crack problem in composite laminates. They used beam and interface methodologies to obtain crack behavior in the steady-state condition. Cheng et al. [19] investigated the problem of a finite crack with constant length propagating in a functionally graded coating bonded to a homogeneous substrate under anti-plane loading.

In the most of these studies, the interfaces between the coating and the substrate are often assumed to be perfect; however, the assumption of perfect bonding is sometimes inadequate. Bagheri et al. [20] solved the anti-plane shear problem of orthotropic strips with multiple stationary cracks and imperfect FGM coating. They studied the effects of material properties of the FGM layer and the spring constant of imperfect boundary on the stress fields. They have employed the distributed dislocation technique for this purpose. This technique is capable of the analysis of multiple defects. This static analysis is aimed to be extended to the moving crack in this paper.

The main objective of this study is to apply the distributed dislocation technique for the stress analysis of multiple moving cracks with arbitrary arrangement in a graded orthotropic layer imperfectly bonded to an orthotropic substrate. The elastic stiffness constants and mass density of materials are assumed to vary exponentially perpendicular to the direction of the crack propagation. The Galilean transformation is employed to express the wave equations in terms of coordinates that are attached to the moving crack. Numerical calculations are carried out for the effects of the crack speed, non-homogeneity parameters and bonding coefficient on the stress intensity factors.

2. Dislocation solution

Consider an orthotropic homogeneous substrate of thickness h_2 bonded to a functionally graded orthotropic coating of thickness h_1 (Fig. 1). In multiple cracks problems, the distributed dislocation technique is often used for treating cracks with smooth geometries [21]. This method relies on the knowledge of stress field due to a single dislocation in the region of interest. However, determining stress fields due to a single dislocation in the region has been a major obstacle to the utilization of this method. We now take up this task for an orthotropic strip with imperfect functionally graded orthotropic coating containing a moving screw dislocation. For the anti-plane analysis, there is only out-of-plane displacement W in each elastic layer, and other two elastic displacements u and v oriented in the x- and y-axes vanish. Precisely, in the homogeneous elastic layers, the basic equations which govern the anti-plane deformation of the layers can be expressed in a fixed Cartesian coordinate system (X, Y) as

$$G_X(Y)\frac{\partial^2 W}{\partial X^2} + G_Y(Y)2\lambda\frac{\partial W}{\partial Y} + G_Y(Y)\frac{\partial^2 W}{\partial Y^2} = \rho_1(Y)\frac{\partial^2 W}{\partial t^2} \quad 0 < Y < h_1,$$
$$\mu_X\frac{\partial^2 W}{\partial X^2} + \mu_Y\frac{\partial^2 W}{\partial Y^2} = \rho_0\frac{\partial^2 W}{\partial t^2} \quad -h_2 < Y < 0. \tag{1}$$

where $G_X(Y)$ and $G_Y(Y)$ are material constants of FGM orthotropic layer, and parameters μ_X and μ_Y are the shear moduli of elasticity of the orthotropic strip in the x- and y-directions, respectively. It should be noted that body forces are neglected in the present work. For simplicity, we will assume that the shear moduli $G_X(Y), G_Y(Y)$ and mass density $\rho_1(Y)$ of functionally