Applied Mathematics and Computation 293 (2017) 438–447

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

# Observer-based periodically intermittent control for linear systems via piecewise Lyapunov function method

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ARTICLE INFO

*Keywords:*
Periodically intermittent control
Observer
Stabilization
Piecewise Lyapunov function

ABSTRACT

This paper focuses on the observer-based periodically intermittent control issue for linear systems by utilizing piecewise Lyapunov function method. Based on the concept of intermittent control, the description of an observer-based periodically intermittent controller is initially proposed. Then, stability of the corresponding periodically intermittent control system is analyzed by resorting to piecewise Lyapunov function method. Here, the so-called piecewise Lyapunov function means that Lyapunov function on control time intervals differs from the one on free time intervals. Besides, the existence of an observer-based periodically intermittent controller is converted into the feasibility of linear matrix inequalities. It is worth pointing out that, in comparison with an observer-based continuous controller, the designed observer-based periodically intermittent controller can still retain satisfactory performance with shorter control task execution time. Finally, an illustrative simulation example is given to show the validity and superiority of the result.



## 1. Introduction

Intermittent control, as a transition between continuous control and impulsive control, emerges in control field. Its characteristic lies in that the control signal is imposed on a plant during certain nonzero time intervals, but is off during other nonzero time intervals. Due to its convenient implementation and high efficiency, intermittent control has been applied to various fields such as secure communication, medical treatment, air-quality control, transportation, and so on.

Recently, much attention has been paid to intermittent control technique to investigate two types of problems: stabilization (see references [1–8]) and synchronization (see references [9–17]). For the former, the description of a periodically intermittent state feedback controller is introduced in [1], based on which an exponential stability criterion of a periodically intermittent control system is obtained by using Lyapunov function theory. Paper [3] studies the exponential stabilization for a class of delayed chaotic neural networks by periodically intermittent control, and the exponential stabilization criterion is established by Lyapunov function and Halanay inequality. In [5], a periodically intermittent state feedback controller is designed to stabilize a class of uncertain nonlinear time-delay systems, and Lyapunov–Krasovskii functional method is adopted to seek the exponential stabilization conditions. For the latter, paper [9] focuses on the synchronization of chaotic systems via intermittent feedback method, and a Lyapunov function is constructed to deduce a synchronization criterion. In [13], the complete synchronization of chaotic neural networks with time delays is explored by intermittent control with

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http://dx.doi.org/10.1016/j.amc.2016.08.042
0096-3003/

two switches in a control period, and Lyapunov stability theory is used to derive some sufficient conditions for complete synchronization. Paper [15] deals with the exponential synchronization for chaotic systems with time delay by means of periodically intermittent state feedback control, and Lyapunov–Razumikhin methodology is utilized to deduce a new exponential synchronization criterion.

Whether for the former or the latter, Lyapunov function or functional plays a critical role during the procedure of derivation. However, it should be observed that, in above literatures, the constructed Lyapunov function or functional on control time intervals is the same with the one on free time intervals, which leads to more conservativeness of the deduced results. This inspires the authors to adopt piecewise Lyapunov function or functional to analyze the stability of a periodically intermittent control system. The main idea of piecewise Lyapunov function or functional discussed in literatures [18–21] is to construct different Lyapunov functions or functionals on time interval sequence. In comparison with Lyapunov function or functional, piecewise Lyapunov function or functional overcomes the limitation of Lyapunov function or functional and presents more generality. Therefore, in this paper, Lyapunov function is replaced by piecewise Lyapunov function to establish less conservative stability and stabilization criteria. Here, the so-called piecewise Lyapunov function means that Lyapunov function on control time intervals differs from the one on free time intervals.

On the other hand, for the stabilization issue, periodically intermittent state feedback controllers are usually considered to stabilize the concerned systems. However, in most practical situations, system states are unmeasurable. Literatures [22–26] do much work on this issue. In [22], an observer-based state feedback controller is addressed to stabilize a class of nonlinear time-delay systems subjected to input and output time-varying delays. Delay-dependent observer-based $H_\infty$ finite-time control problem is explored in [24] for switched systems with time-varying delay. It should be pointed out that these papers focus on the design problem of an observer-based state feedback controller and do not involve in an observer-based periodically intermittent control issue.

Motivated by above two aspects, we are concerned with the observer-based periodically intermittent control in this paper for linear systems by utilizing piecewise Lyapunov function method. The main contributions of this paper lie in the following three aspects. Firstly, based on the concept of intermittent control, the description of an observer-based periodically intermittent controller is proposed. Secondly, piecewise Lyapunov function is constructed to analyze the stability of the corresponding periodically intermittent control system. The so-called piecewise Lyapunov function means that Lyapunov function on control time intervals differs from the one on free time intervals. Thirdly, the existence of an observer-based periodically intermittent controller is converted into the feasibility of linear matrix inequalities. Compared with an observer-based continuous controller, the designed observer-based periodically intermittent controller is still capable of retaining satisfactory effects with shorter control task execution time.

**Notation:** In this paper, notations are fairly standard. $R^n$ denotes the $n$ dimensional Euclidean space, $\|\cdot\|$ refers to the Euclidean vector norm, and $R^{n\times m}$ represents the field of $n\times m$ dimensional real matrices. The superscripts $T$ and $-1$ stand for the transposition and the inverse of a matrix, respectively. Matrix $A>0(<0, \geq 0, \leq 0)$ means that $A$ is positive definite (negative definite, positive semi-definite, negative semi-definite). $\star$ stands for the symmetric term in a symmetric matrix. In addition, $\lambda_m(P)(\lambda_M(P))$ denotes the minimum (maximum) eigenvalue of matrix $P$.

## 2. Problem formulation and preliminaries

Consider the following linear system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \quad t > 0, \\ y(t) = Cx(t), \\ x(0) = x_0, \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $y(t) \in R^q$ is the measured output. $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{q\times n}$ are the known real constant matrices. $x_0$ is the initial state vector.

**Assumption 1.** When $u(t)=0$, system (1) is unstable.

**Assumption 2.** The state vector $x(t)$ is unmeasurable.

**Assumption 3.** $(A, B)$ is completely controllable and $(A, C)$ is completely observable.

**Assumption 4.** Matrix $C$ is of full row rank.

Here, the description of an observer-based periodically intermittent controller is proposed as

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + L(t)(y(t) - \hat{y}(t)) + Bu(t), \\ \hat{y}(t) = C\hat{x}(t), \\ \hat{x}(0) = \hat{x}_0, \\ u(t) = K(t)\hat{x}(t), \end{cases} \tag{2}$$