Applied Mathematics and Computation 293 (2017) 565–574

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Binary Bell polynomials, Hirota bilinear approach to Levi equation

Yaning Tang*, Weijian Zai, Siqiao Tao, Qing Guan

Department of Applied Mathematics, Northwestern Polytechnical University, Xi'an 710129, PR China

ARTICLE INFO

Keywords:
Binary Bell polynomials
Double Wronskian solutions
Bäcklund transformation and Lax pair
Darboux transformation
Infinite conservation laws

ABSTRACT

Combining the binary Bell polynomials and Hirota method, we obtained two kinds of equivalent bilinear equations for the Levi equation. Then, we got the double Wronskian solutions of the Levi equation by virtue of one of the bilinear equations. Furthermore, we constructed the bilinear Bäcklund transformation and the Lax pair. Finally, we also derived the Darboux transformation and the infinite conservation laws of the Levi equation.

1. Introduction

Nonlinear equations are widely used in the fields of physics, chemistry, atmospheric dynamics, life sciences and engineering sciences etc. There have been many great researches on the nonlinear equations in recent years. Particularly in soliton theory, many different approaches can be used to deal with the nonlinear equations. Inverse scattering transformation [1], Bäcklund transformation [2], Darboux transformation [3], Hirota bilinear method [4,5] and Wronskian technique [6] are all very cogent methods. Among those methods, the bilinear method developed by Hirota is powerful for analyzing soliton equations. The first step of this method is to transform the given soliton equations into bilinear forms by proper dependent variables transformation. By means of the bilinear representations for soliton equations, not only the multi-soliton solutions but also the main integrability features of these equations such as bilinear Bäcklund transformations, Lax pairs and Darboux transformation can be derived. However, the well chosen dependent variables transformation for deriving the bilinear forms cannot be obtained easily, because it relies on clever guesswork and tedious calculation. Recently, Gilson et al. [7] and Lembert and Springael [8,9] found there exists a deep connection between the Bell polynomials [10] and Hirota's bilinear expressions and then proposed a lucid and systematic method to overcome these difficulties. So both the bilinear forms and bilinear Bäcklund transformations can be obtained directly. Lots of studies have devoted to the application and extension of this method [11–21].

In this paper, we will study the Levi equation

$$u_t - 2uu_x - 2v_x + u_{xx} = 0, \tag{1.1a}$$

$$v_t - 2(uv)_x - v_{xx} = 0, \tag{1.1b}$$

Eqs. (1.1) have been investigated in [22] for solving the double Wronskian solutions. However, the bilinear Bäcklund transformations, Lax pair, Darboux transformation as well as the infinite conservation laws for Eqs. (1.1) have not been studied based on the binary Bell polynomials and Hirota method.

* Corresponding author.
E-mail address: tyaning@nwpu.edu.cn (Y. Tang).

http://dx.doi.org/10.1016/j.amc.2016.08.022

On account of the above analysis, this paper is organized as follows. In Section 2, we will briefly introduce concepts and formulae about the binary Bell polynomials. In Section 3, we will derive two kinds of equivalent bilinear equations for Eqs. (1.1), then we will get the double Wronskian solutions of Eqs. (1.1). In Section 4, we will give the bilinear Bäcklund transformation and the corresponding Lax pair. In Section 5, we will obtain the Darboux transformation by introducing a gauge transformation. In Section 6, we also will get the infinite conservation laws of Eqs. (1.1). Finally, we will draw some conclusions.

2. The binary Bell polynomials

Let $f = f(x_1, \ldots, x_l)$ is a C^∞ function of multi-variables, the definition of multi-dimensional Bell polynomials (see [7–10] for details) reads as

$$Y_{n_1x_1,\ldots,n_lx_l}(f) \equiv Y_{n_1,\ldots,n_l}(f_{r_1x_1,\ldots,r_lx_l}) = e^{-f}\partial_{x_1}^{n_1}\ldots\partial_{x_l}^{n_l}e^f, \tag{2.1}$$

where $f_{r_1x_1,\ldots,r_lx_l} = \partial_{x_1}^{r_1}\ldots\partial_{x_l}^{r_l}f,\ r_1 = 0,\ldots,n_1;\ \ldots;\ r_l = 0,\ldots,n_l$.

Based on the above Bell polynomials, when $l = 2$ in (2.1), the binary Bell polynomials ($\mathcal{Y}$-polynomials) can be defined by

$$\mathcal{Y}_{n_1x,n_2t}(v,w) = Y_{n_1x,n_2t}(f)\Big|_{f_{r_1x,r_2t}=\begin{cases} v_{r_1x,r_2t}, & r_1+r_2=\text{odd},\\ w_{r_1x,r_2t}, & r_1+r_2=\text{even}.\end{cases}} \tag{2.2}$$

For example, the first few lowest order $\mathcal{Y}$-polynomials are

$$\begin{aligned}\mathcal{Y}_{x,t}(v,w) &= w_{xt} + v_xv_t,\\ \mathcal{Y}_{2x,t}(v,w) &= v_{2x,t} + w_{2x}v_t + 2w_{xt}v_x + v_x^2v_t,\\ \mathcal{Y}_{3x}(v,w) &= v_{3x} + 3w_{2x}v_x + v_x^3, \ldots .\end{aligned}$$

The link between $\mathcal{Y}$-polynomials and Hirota's standard bilinear expressions

$$D_x^{n_1}D_t^{n_2}F\cdot G = (\partial_x - \partial_{x'})^{n_1}(\partial_t - \partial_{t'})^{n_2} \times F(x,t)G(x',t')\,|_{x'=x,t'=t} \tag{2.3}$$

is given by the identity

$$\mathcal{Y}_{n_1x,n_2t}(v = \ln F/G, w = \ln FG) = (FG)^{-1}D_x^{n_1}D_t^{n_2}F\cdot G. \tag{2.4}$$

Particularly, in the case of $F = G$, we have

$$\begin{aligned}F^{-2}D_x^{n_1}D_t^{n_2}F\cdot F &= \mathcal{Y}_{n_1x,n_2t}(0, w = 2\ln F)\\ &= \begin{cases}0, & n_1+n_2=\text{odd},\\ P_{n_1x,n_2t}(w), & n_1+n_2=\text{even},\end{cases}\end{aligned} \tag{2.5}$$

which discloses that P-polynomials can be obtained by restricting the Bell polynomials to even part partitions.

Moreover, there has a relationship between the binary Bell polynomials and Lax pair through the following expression

$$\begin{aligned}&\mathcal{Y}_{n_1x,n_2t}(v = \ln\Psi, w = v + Q)\\ &= \Psi^{-1}\sum_{a_1=0}^{n_1}\sum_{a_2=0}^{n_2}\binom{n_1}{a_1}\binom{n_2}{a_2}P_{a_1x,a_2t}(Q)Y_{(n_1-a_1)x,(n_2-a_2)t}(v = \ln\Psi),\end{aligned} \tag{2.6}$$

where Ψ and Q are functions of x and t.

3. Bilinear equations and double Wronskian solutions

In this section, with the help of the binary Bell polynomials, we will give two kinds of equivalent bilinear forms of Eqs. (1.1).

Let

$$u = p_x, \quad v = \frac{1}{2}(q_{xx} + p_{xx}), \tag{3.1}$$

Eqs. (1.1) are transformed to

$$p_t - (p_x^2 + q_{xx}) = 0, \tag{3.2a}$$

$$q_{xt} - p_{xxx} - 2p_xq_{xx} = 0, \tag{3.2b}$$

using Eqs. (3.2a), Eqs. (3.2) are further transformed to

$$p_t - (p_x^2 + q_{xx}) = 0, \tag{3.3a}$$