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Mittag–Leffler stability analysis of nonlinear fractional-order systems with impulses

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ABSTRACT

This paper is designed to deal with the Lyapunov stability analysis of fractional-order nonlinear systems with impulses. Based on the theory of fractional calculus, impulsive differential equation and S-procedure, several sufficient criteria are established to guarantee the Mittag–Leffler stability for the addressed model with appropriate impulsive controller. Furthermore, two numerical examples are given to verify the validity and feasibility of the obtained results.

1. Introduction

The development of the theory of the fractional calculus can be dated back to 17th century along with the classical inter-order calculus. Unfortunately, the application of fractional calculus did not gain adequate attention until specialists and scholars declare that fractional derivatives possess the superior merits of memory and hereditary properties [1–3]. In recent decades, fractional derivative has been applied to model mechanical and electrical properties of various real materials, meanwhile, fractional derivative has also been extensively incorporated into all kinds of dynamical systems, for example, see [4–9] and reference therein.

In recent years, some scholars try their efforts to extend the theory of classical impulsive differential equation [10] to the case of fractional order, and the dynamical behaviors of impulsive evolution models of fractional order have become an active research topic. On the basis of Mittag–Leffler stability theory and the Lyapunov direct method proposed by Podlubny and his colleagues [11,12], a considerable number of results of stability analysis for fractional impulsive dynamical systems have been reported, see [13–16] and reference therein. In [13,14], the authors proposed a series of conclusions on the asymptotically stable and Mittag–Leffler stable for impulsive fractional differential equations by applying Lyapunov direct method. In [15], the authors established a Lyaponov function with the term of Riemann–Liouville operator, and investigated the asymptotic stability for fractional network models. In [16], on account of the graph theory and Lyapunov method, the authors discussed the asymptotic stability and Mittag–Leffler stability for a class of feedback control systems of fractional differential equations on networks with impulsive effects. It should be recognized that in [17], the authors presented a general quadratic Lyapunov function which has been employed to establish some stability criteria for many fractional systems

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effectively, for example, see [18,19]. As far as we know, there exists few result on the stability for nonlinear fractional-order systems with impulsive effects based on general quadratic Lyapunov function.

Motivated by the aforementioned discussions, the objective of this paper is to analyze the stability of a class of fractional-order nonlinear systems with impulses by utilizing the general quadratic Lyapunov function, and several sufficient conditions equipped with the terms of linear matrix inequalities will be presented.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n \times n$ real matrices, respectively. I stands for the identity matrix with appropriate dimension. Q^T means the transpose of matrix Q. $\|u\| = \sqrt{u^T u}$ stands for the Euclidean norm of a real vector u. For a real matrix A, $\lambda_{max}(A)$ and $\lambda_{min}(A)$ denote, respectively, the maximal and minimal eigenvalues of A, and $A > 0(< 0)$ means the matrix A is symmetric and positive definite (or negative definite). In addition, let $\mathbb{R}_+ = [0, +\infty)$, $\mathbb{Z}_+ = \{1, 2, 3, \ldots\}$, Ω be an open set in $\mathbb{R}^n$ containing the origin, $G_k = (t_{k-1}, t_k) \times \Omega$, $k \in \mathbb{Z}_+$, and $G = \bigcup_{k=1}^{\infty} G_k$.

2. Model description and preliminaries

In this paper, we consider the following fractional-order nonlinear systems with impulses:

$$\begin{cases} D_{0,t}^{\alpha} u(t) = Au(t) + g(t, u(t)), & t \neq t_k, \\ \Delta u(t_k) = u(t_k^+) - u(t_k^-) = J_k(u(t_k)), & t = t_k, \ k \in \mathbb{Z}_+. \end{cases} \tag{1}$$

where $D_{0,t}^{\alpha}$ denotes the Caputo fractional derivative of order α, $(0 < \alpha < 1)$, $u(t) \in \mathbb{R}^n$ is the state vector, $A \in \mathbb{R}^{n\times n}$ is a constant matrix, $g(t, u(t)) \in \mathbb{R}^n$ is the nonlinear term with $g(t, 0) = 0$, $J_k(\,\cdot\,)$ stands for the jump operator of impulsive, and the impulsive moments satisfy $0 = t_0 < t_1 < t_2 < \cdots < t_k < t_{k+1} < \cdots$ with $\lim_{k\to+\infty} t_k = \infty$.

Let $u_0 \in \Omega$. Denote by $u(t) = u(t, t_0, u_0)$ the solution of (1), satisfying the initial condition

$$u(t_0, t_0, u_0) = u_0, \tag{2}$$

and in order to ensure the existence, uniqueness and continuability of the solution $u(t) = u(t, t_0, u_0)$, we further assume that each solution $u(t)$ of (1) is piecewise continuous with points of discontinuity of the first type at which they are left-continuous, that is, $\lim_{t\to t_k^-} u(t) = u(t_k)$.

In the following, we first introduce some definitions of fractional calculus.

The α-order Riemann–Liouville fractional integral and Caputo fractional derivative for a suitable function are defined as, respectively

$$D_{0,t}^{-\alpha} u(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} u(s) ds, \quad n-1 < \alpha < n, \tag{3}$$

$$D_{0,t}^{\alpha} u(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-s)^{n-\alpha-1} u^{(n)}(s) ds, \quad n-1 < \alpha < n. \tag{4}$$

The one-parameter Mittag–Leffler functions is defined as

$$E_{\alpha}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(\alpha k + 1)}, \quad \alpha > 0. \tag{5}$$

Throughout this paper, we make the following assumption.

(H) The function $g(t, u) : \mathbb{R}_+ \times \Omega \to \mathbb{R}^n$ is Lipschitz continuous on u with $g(t, 0) = 0$, that is, for all $t \in \mathbb{R}_+$, there exists a positive number ζ such that

$$\|g(t, u) - g(t, v)\| \le \zeta \|u - v\|,$$

for all $u, v \in \mathbb{R}^n$.

In order to obtain our main results, the following definitions and lemmas are requisite.

Definition 1 [10]**.** The function $V : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}_+$ is said to belong to class V_0 if

(1) $V(t, u)$ is continuous in G_k, $k \in \mathbb{Z}_+$, and for each $u \in \mathbb{R}$, $\lim_{(t,v)\to(t_k^+,u)} V(t, v) = V(t_k^+, u)$ exits, and $V(t_k^-, u) = V(t_k, u)$;
(2) $V(t, u)$ is locally Lipschitz continuous with respect to u.

Definition 2 (Mittag–Leffler stability) [11]**.** The solution of system (1) is said to be Mittag–Leffler stable if

$$\|u(t)\| \le \left\{ m(u(t_0)) E_{\alpha}(-\lambda(t-t_0)^{\alpha}) \right\}^{\vartheta}, \tag{6}$$

where t_0 is the initial time, $\alpha \in (0, 1)$, $\lambda > 0$, $\vartheta > 0$, $m(0) = 0$, $m(u) \ge 0$, and $m(u)$ is locally Lipschitz on $u \in \mathbb{R}$ with Lipschitz constant m_0.

Remark 1. Mittag–Leffler stability implies asymptotic stability. By applying the Lyapunov direct method, the asymptotic stability of the corresponding systems can be obtained. Furthermore, if we extend the Lyapunov direct method to the case of fractional-order systems, the Mittag–Leffler stability for such concerned systems could be established.