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Attenuation of dissipative device involving coupled wave scattering and change in material properties

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ABSTRACT

This paper deals with the attenuation and performance of dissipative device like hybrid silencer involving coupled wave scattering for two different mode incidents. In particular device comprises inlet/outlet duct sections and a membrane attached internally to the outlet duct which may vary to tune the device. The solution process using the hybrid mode matching technique insight the physical situation of the underlying problem. The power distribution between the fluid regions and the membrane(s) is discussed numerically. We also confirm the validation of power balance as a fundamental property of the truncated system.

1. Introduction

The study of non-uniform obstacles in an otherwise uniform waveguide has received wide attention in the literature. The transmission of elastic and electromagnetic waves, underwater sound propagation, and sound scattering in ducts or pipes are the major applications to such studies. The curiosity is to reduce the ducted fan noise aero engines, power stations and heating, ventilation, and air conditioning (HVAC) systems. Numerous investigations have been made to suggest analysis for the reduction of unwanted noise from different types of obstacles [1–6] There are several mathematical models exist for computing sound attenuation by dissipative devices, see for example [7–10] Such devices are often used to attenuate broadband noise arising from fluid moving devices likewise fans and internal combustion engines. Since the dissipative devices neglect sound scattering over the inlet and outlet planes of the silencer therefore such models have a limited scope for predicting overall silencer performance.

Generally, measurements of silencer performance are reported in terms of the sound power difference across the silencer. Keeping this in view, Cummings and Chang [11] imposed continuity of pressure and velocity over the inlet and outlet planes of the silencer while using the silencer eigenmodes in an analytic mode matching scheme. Peat [12], and later by Kirby [13], proposed closed form analytic solutions based on the attenuation of the fundamental mode alternative to Cummings and Chang's method. Again the procedure opted by Peat [12] and Kirby [13] were only valid for a restricted frequency range. Such limitations apply also to other methods based on the fundamental mode, for example those methods suggested by Panigrahi and Munjal [14]. Recently Afzal et al. [15] and, Nawaz et al. [16], incorporated the second mode forcing term that aim to carry energy through the fluid.

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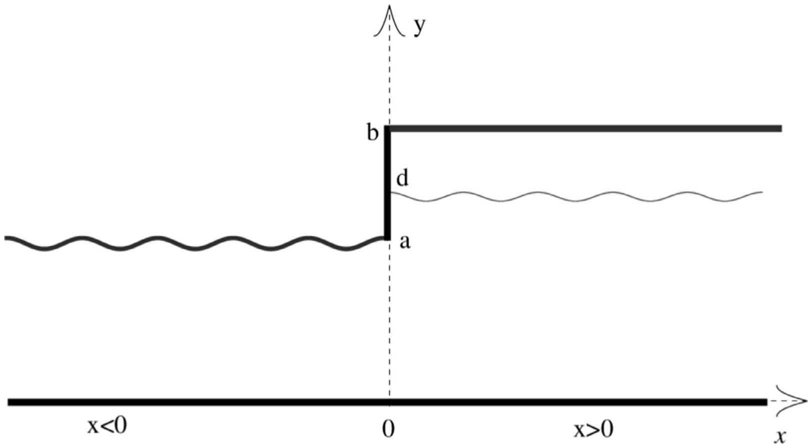

Fig. 1. The geometrical configuration of the problem.

This paper investigates the study of a device comprises a two dimensional reactive silencer in which a membrane is attached to the internal walls of the expansion chamber and an elastic plate is attached to the outlet duct parallel to the axis of the inlet/outlet ducts. The height of the membrane above the level of the inlet/outlet ducts can be varied, and the device is tuned by selecting the membrane height that gives the widest stopband for a specified frequency. Dowell and Voss [17] studied the vibrations of a cavity backed panel in the presence of flow by familiarizing with the concept of flexible panels. In a recent study, Kang and Fuchs [18] have investigated the effectiveness of cavity-backed microperforated membranes as an acoustic absorber. While working through different structural acoustic models, diverse analytic and numerical techniques are used to obtain the solution. Different transforms and numerical approaches have widely been examined to study wavelike equation such as [19,20]. Miles [21] and later by Selamet and Easwaran [22] established plane wave propagation in variable area ducts; whereas Bostrom [23] employed analytic techniques to undertake scattering by spherical and spheroidal obstacles in a duct.

With all above this work initiates by finding the eigenmodes in the silencer chamber and the inlet/outlet ducts. The mathematical procedure contains the use of well-known mode-matching technique. The technique and related structures has been studied by many authors [21–23] to demonstrate different physical situations. We aim to investigate the underlying problem by the effects of varying the height of the membrane which is attached internally to the outlet duct; to discuss the potential use of this device as a component of a hybrid silencer for HVAC ducting systems; to study propagation of reflected and transmitted energy fluxes by varying duct heights, to validate the physical insight of the problem and the MM technique used herein. In this process the paper is planned in the following manners: The traveling wave form for the purposed configuration is stated in Section 2 whereas MM solution together with the appropriate OR is established in Section 3. Section 4 comprises the expressions for power distribution which are then evaluated numerically in Section 5. The investigation is then summarized in Section 6.

2. Problem statement

This section comprises of a boundary value problem describing the sound attenuation of dissipative silencer through a semi infinite duct. Let us consider a duct configuration occupying the region $\bar{x} < 0, 0 < \bar{y} < \bar{a}$ and $\bar{x} > 0,\ 0 < \bar{y} < \bar{b}$, where $(\bar{x}, \bar{y})$ are dimensional Cartesian coordinates. The interior region of the waveguide is filled with compressible fluid of density ρ and sound speed c. The duct sections comprise acoustically rigid surfaces at $\bar{y} = 0, -\infty < \bar{x} < \infty$ and $\bar{y} = \bar{b},\ 0 < \bar{x} < \infty$. At $\bar{y} = \bar{a}$, $\bar{x} < 0$ the left hand duct that is inlet duct, is bounded by membrane surface whilst at $\bar{y} = \bar{d}$, $\bar{x} > 0$ in the right hand duct, that is outlet duct, a horizontal membrane is located which divide the fluid in two regions. The upper surfaces of inlet duct and outlet duct are joined by means of rigid vertical strip lying at $\bar{x} = 0,\ \bar{a} < \bar{y} < \bar{b}$. It is mentioned that the quantities with bar notations such as $\bar{a}$,$\bar{b}$ and $\bar{d}$ are the dimensional values specifying position of each surface. Whereas these quantities without bar notations such as a, b and d are dimensionless. The model duct configuration is shown in Fig. 1.

On assuming the harmonic time dependence $e^{-i\omega\bar{t}}$, where $\omega = ck$ is the radian frequency in which k is fluid wave number and c is the sound speed, the velocity potential $\bar{\Psi}(\bar{x}, \bar{y}, \bar{t})$ in duct regions can be expressed in terms of time independent velocity potential $\bar{\Psi}(\bar{x}, \bar{y}, \bar{t}) = e^{-i\omega\bar{t}}\bar{\psi}(\bar{x}, \bar{y})$. On non-dimensionalizing with respect to length scale k^{-1} and time scale ω^{-1} under the transformation $x = k\bar{x}$ and $y = k\bar{y}$ etc., the non-dimensional form of Helmholtz's equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 1\right)\psi(x, y) = 0. \tag{1}$$