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Optimal contracts for central bankers: Calls on inflation

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ABSTRACT

We consider a framework featuring a central bank, private and financial agents as well as a financial market. The central bank's objective is to maximize a functional, which measures the classical trade-off between output and inflation plus income from the sales of inflation linked calls minus payments for the liabilities that the inflation linked calls produce at maturity. Private agents have rational expectations and financial agents are averse against inflation risk. Following this route, we explain demand for inflation linked calls on the financial market from a no-arbitrage assumption and derive pricing formulas for inflation linked calls, which lead to a supply–demand equilibrium. We then study the consequences that the sales of inflation linked calls have on the observed inflation rate and price level. Similar as in Walsh (1995) we find that the inflationary bias is significantly reduced, and hence that markets for inflation linked calls provide a mechanism to implement inflation contracts as discussed in the classical literature.

1. Introduction

Inflation contracts have been widely discussed in the macro-economic and monetary policy literature. In principal agent manner, the central banker is offered a compensation, which depends on the realized inflation level. This approach was developed by Persson and Tabellini [7] as well as Walsh [13] and shown to be a useful device in order to remove inflationary bias, and in this way raise general welfare. The contracts approach was further discussed in Svensson [12] and Muscatelli [6], and put into relation with the concept of inflation targeting. Muscatelli highlighted some advantages of the contracts approach as compared to the inflation targeting approach, in the context of uncertainties in preferences and output targets.

The contracts proposed by Persson and Tabellini [7] and Walsh [13] are between the government acting as a principal and the central banker as an agent. In this note we show that this principal agent relationship is not required, and that financial markets for inflation indexed securities can reduce inflationary bias in the same way as the inflationary contracts discussed in the classical literature. Rather than forcing the central banker to enter an inflation contract with the government, we allow the central banker to sell inflation contracts in form of inflation linked calls to financial agents. The central banker can choose the quantity of contracts placed on the market and in this way determines the supply. Demand is mainly determined by the financial agents' level of risk aversion toward inflationary risk and their expectations about the central banks inflation policy. The type of contract proposed in this paper is realistic, in fact, according to Deacon et al. [2] no less than 27% of UK government debt is inflation indexed and treasury inflation protected securities (TIPS) issued by the US Treasury contain as their final payment an inflation linked call.

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The model that we present is a full general equilibrium model, in which quantity and price of the inflation linked calls is determined by equating supply and demand, and private as well as financial agents reflect rational expectations in their inflation forecast. In this latter aspect, the current paper differs structurally form Ewald and Geissler [3], who assumed adaptive expectations (as well as a different contract structure). The supply side is modeled in analogy with Walsh [13] and most of the other inflation contract literature, with the difference that the inflation contract is replaced by a quantity of inflation linked calls, which the central bank can choose. The demand side is modeled in analogy to Black–Scholes, assuming that the price level follows a geometric Brownian motion, whose drift rate is the inflation level chosen by the central bank.

While the identification of markets for inflation linked securities as a Walsh [13] like mechanism contributes to the economic literature, our results also contribute to the financial literature. Almost all of the classical financial literature on pricing of inflation indexed securities takes inflation as exogenously given, and applies standard Black–Scholes type theory or the Heath–Jarrow–Morton approach to term structure modeling in order to develop pricing formulas, see for example Deacon et al. [2] and Jarrow and Yilderim [4]. These models ignore the central bank's role in issuing inflation indexed securities and the feedback effects on monetary policy this has. To the best of our knowledge, this paper presents the first pricing formulas for inflation linked calls in which monetary policy is integrated within a full general equilibrium framework.

2. Central bank's supply of inflation linked calls

An inflation linked call issued at time s with strike $\tilde{K}$ and maturity T is a financial derivative that pays off nominal

$$\left(\frac{P_T}{P_s} - \tilde{K}\right)^+ = \left(e^{\pi(T-s)} - e^{K(T-s)}\right)^+ := \max\left(e^{\pi(T-s)} - e^{K(T-s)}, 0\right) \tag{1}$$

at maturity time T. Here we have set $K = \frac{1}{T-s}\log(\tilde{K})$. Deacon et al. [2] present an excellent overview about all types of traded inflation indexed securities. Here P_t denotes the price level at time t and π denotes the average instantaneous inflation rate over time $T-s$. We consider a simplistic setup, in which the central bank can issue inflation linked calls at time $s=0$ which mature at time $T=1$. Denoting log-output with y, natural log-output with y_n and a target value with k, the classic quadratic loss function of the central bank has to be modified in the following way

$$Z^{CB} := \frac{1}{2}\left[\lambda(y - y_n - k)^2 + \pi^2\right] + N \cdot d\left[\left(e^{\pi} - e^{K}\right)^+ - p\right], \tag{2}$$

in order to take account of the central bank's profits from the sales of inflation linked calls. Here p denotes the price of one inflation linked call and N the number of inflation linked calls issued by the central bank. The factor d is a weight and measures the contribution of the financial position of the central bank in relations to its output and inflation objectives. Expression (2) is of similar type as expressions (6) and (7) in Muscatelli [6], which include a penalty function and an exogenously defined inflation contract instead of the profits from inflation linked call sales. However, in contrast to Muscatelli and all other literature, in our case the price p and the quantity N, and as such the inflation contract itself will be determined endogenously within the model, via the financial market modeled in the next section.

As in the classical literature we assume that output is given by a Lucas-type aggregate supply function of the form

$$y = y_n + a(\pi - \pi^e) + \varepsilon, \tag{3}$$

with a being the slope of the Phillips curve, π^e expected inflation and ε a stochastic shock to the economy with zero mean under the central bank's measure. We assume in the following that $\epsilon \sim \mathcal{N}(0, \sigma)$.[1]

We assume that the central bank is able to observe the economy shock ε, but private agents are not.[2] This means that the central bank's inflation policy can depend on ε, i.e. $\pi = \pi(\epsilon)$, while private agents' expectations must not. Substitution of (3) into (2) and taking expectations gives

$$V := \mathbb{E}\left\{\frac{1}{2}\lambda[a(\pi - \pi^e) + \epsilon - k]^2 + \frac{1}{2}\pi^2 + N \cdot d\left[\left(e^{\pi} - e^{K}\right)^+ - p\right]\right\}. \tag{4}$$

The quantity N and price p of inflation linked calls will be determined in full equilibrium in Section 4. For given N and p we will now compute the central bank's optimal inflation policy. To stress that this policy in general depends on N we write $\pi(N)$ in the following. We will later look for a rational expectation equilibrium, where $\pi^e = \mathbb{E}(\pi)$, and as π depends on N, so will π^e. Hence we will include N in the notation for the expected inflation rate as well, and assume for the moment that $\pi^e = \pi^e(N)$ is given.

Proposition 1. *The central bank's optimal choice for the inflation rate $\pi(N)$ as a function of the shock ε is given by*

$$\pi^*(N, \varepsilon) = \begin{cases} \xi(N, \varepsilon) & \text{if} \quad \varepsilon \geq \eta(N) \\ K & \text{if} \quad \theta(N) \leq \varepsilon \leq \eta(N) \\ \psi(N, \varepsilon) & \text{if} \quad \varepsilon \leq \theta(N), \end{cases}$$

[1] For a more general discussion on noise, please compare Sun et al. [8,9] as well as Li and Jin [5].

[2] In a more dynamic version of the model private agents would observe the economy shock with a time delay. How to deal with time delay conceptually is discussed in Sun et al. [10,11].