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Pattern dynamics of a spatial epidemic model with time delay

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ABSTRACT

The nonlinear incidence rate can explain the complicated infectious process of disease. And time delay describing the latent period widely exists in the process of disease contagion. In this paper, a spatiotemporal epidemic model with nonlinear incidence rate is investigated. In particular, we considered that the time delay is relatively small. In this case, the characteristic equation are derived, we obtain two mechanisms of instability of the positive constant stationary state, that is, One is the diffusion induced instability, and the other one is delay induced instability. Moreover, the results of numerical simulation validate our theoretical analyses. The obtained results may well catch some major features for epidemic models.

1. Introduction

At present, more and more diseases spread to many areas from source areas. For example, at the end of 3 2009, H1N1 avian influenza began to emerge in Mexico and the United States, and then spread to all over the world. The prevention and control of the disease is an urgent issue to be solved [1–5]. On the basis of Kermack–McKendrick compartment model, previous researchers have established some epidemic models to study a variety of realistic diseases, such as ordinary differential equation model, partial differential equation model, stochastic differential equation model and so on [6–10]. These observed epidemiological phenomena are explained by modeling suitable models. Su and Ruan have found that malaria fever depended on the parasite replication cycles based on a reaction–diffusion system [11]. Jewell and Keeling et al. have studied the spatiotemporal epidemic model of foot-and-mouth disease of 2007 in the UK, and proposed undetected infections.

All living beings reside in spatial environments, they are not only in one place, and randomly go to the surrounding area. The diffusion of individual in space has an effect on the spread of the disease. For example, populations mobility caused outbreak of the chikungunya epidemic in 2005–2006 on the Reunion Island [12]. In order to study the trends of disease spreading in space, the mechanisms of pattern formation in epidemic models can reflect the evolution and distribution of the infected in space by the reaction–diffusion model [13–20]. Noble used the reaction–diffusion model to understand spreading processes of Black Death in Europe [21]. Thus, epidemic model with respect to space is able to describe the contagion phenomena of disease.

However, an individual infected some disease will not immediately come down with the symptoms, but after a period of time, the individual shows the corresponding symptoms, such as rabies, avian influenza, Ebola and so on [22–25]. In fact, it is possible to promote the outbreak of the disease for these potentially asymptomatic individuals who do not usually go to the hospital or limit their behaviors. The variable periods of latency on the transmission dynamics of tuberculosis affected

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the number of infected individuals [26]. In addition, a few scholars studied the formation of patterns of reaction–diffusion epidemic model with considering the latent period [9,19]. Therefore, it is meaningful to consider the influences of time delay and spatial factor on diseases.

Pattern dynamics has been received more and more attention in various research fields [27,28] and rich dynamics are obtained including phase transitions [29–31] and cyclic dominance [32]. For spatial epidemic models, one can find the regions of diseases outbreak or extinction and take effective control strategies to eliminate the diseases based on patterns structures. What is more, our results on pattern dynamics of epidemic systems in this paper can be applied in other systems, such as social systems [33], ecosystems [34] and so on [35].

Spatial patterns in epidemic systems were investigated by some scholars [9,10,14,16] and they obtained rich patterns formation, such as stripe-like or spotted or coexistence of both patterns. However, little work was done on the pattern dynamics of epidemic models with diffusion and time delay. For such reason, we present a spatial epidemic model with spatial motion and delay in this paper.

The structure of this paper is as below. In Section 2, the local stabilities of equilibria of the non-spatial SI epidemic model are given. In Section 3, in case of small delay, the characteristic equation of spatial SI epidemic model with delay is derived. Moreover, the delay and diffusion can both induce the instability of the positive constant stationary state. In Section 4, we obtain the patterns by performing a series of numerical simulations. Finally, we give some conclusions and discussion.

2. Mathematical modeling and analysis

2.1. Model formulation

The appropriate epidemic model can reflect the situation of the spread of disease. In this paper, the total population is divided into the susceptible (S) and the infectious (I). We assume that only the susceptible can produce offsprings, and the susceptible growth is affected by environmental constraints. Thus, we consider the logistic model. Since the infection process between the susceptible and the infected is more complex, Liu et al. considered nonlinear incidence rate $\beta S^p I^q$ with p and q near 1 and this form of nonlinear incidence rate without a period forcing can induce rich dynamical behaviors [36,37]. The population is always in a certain spatial position, and assuming that the individual randomly diffuse to the surrounding space. In order to describe the diffusion process, the usual Laplacian operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ in two-dimensional space is given. d_1 and d_2 denote the diffusion coefficients for different species. Furthermore, we are interested in the self-organization of patterns, and choose the nonzero initial condition and Neumann boundary conditions. Then the following SI epidemic model is given by:

$$\begin{cases} \dfrac{\partial S(x,y,t)}{\partial t} = rS(x,y,t)\left(1-\dfrac{S(x,y,t)}{K}\right) - \beta(S(x,y,t))^p(I(x,y,t))^q + d_1\nabla^2 S(x,y,t), \\ \dfrac{\partial I(x,y,t)}{\partial t} = \beta(S(x,y,t))^p(I(x,y,t))^q - dI(x,y,t) + d_2\nabla^2 I(x,y,t), t>0, (x,y)\in\Omega, \\ \left.\dfrac{\partial S}{\partial n}\right|_{\partial\Omega} = \left.\dfrac{\partial I}{\partial n}\right|_{\partial\Omega} = 0, \quad t\geq 0, \\ S(x,y,0)\geq 0, \quad I(x,y,0)\geq 0, \quad (x,y)\in\Omega, \end{cases} \tag{1}$$

where r and K are the intrinsic growth rate and the carrying capacity of logistic equation, respectively. The contact transmission rate is β, d denotes the natural mortality. p and q are phenomenological, and $p+q=1$ with $p, q>0$ [9].

Most diseases have incubation [38,39]. For example, human infected H7N9 avian influenza generally takes 7 days to exhibit corresponding symptoms [40]. Therefore, it is meaningful for us to introduce time delay into the infected. Consequently, one can give the following system:

$$\begin{cases} \dfrac{\partial S(x,y,t)}{\partial t} = rS(x,y,t)\left(1-\dfrac{S(x,y,t)}{K}\right) - \beta(S(x,y,t))^p(I(x,y,t-\tau))^q + d_1\nabla^2 S(x,y,t), \\ \dfrac{\partial I(x,y,t)}{\partial t} = \beta(S(x,y,t))^p(I(x,y,t-\tau))^q - dI(x,y,t) + d_2\nabla^2 I(x,y,t), t>0, (x,y)\in\Omega, \\ \left.\dfrac{\partial S}{\partial n}\right|_{\partial\Omega} = \left.\dfrac{\partial I}{\partial n}\right|_{\partial\Omega} = 0, \quad t\geq 0, \\ S(x,y,t)=\phi(x,y,t)\geq 0, \quad I(x,y,t)=\varphi(x,y,t)\geq 0, \quad (x,y,t)\in\bar{\Omega}\times[-\tau,0]. \end{cases} \tag{2}$$

2.2. Local stability

Next, the following non-spatial system is discussed:

$$\begin{aligned} \frac{dS}{dt} &= rS\left(1-\frac{S}{K}\right) - \beta S^p I^q \triangleq f(S,I), \\ \frac{dI}{dt} &= \beta S^p I^q - dI \triangleq g(S,I). \end{aligned} \tag{3}$$