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Approximate controllability of a multi-valued fractional impulsive stochastic partial integro-differential equation with infinite delay

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ABSTRACT

In this paper, the approximate controllability of a new class of multi-valued fractional impulsive stochastic partial integro-differential equations with infinite delay in Hilbert spaces is studied. Firstly, by using stochastic analysis, analytic α-resolvent operator, fractional powers of closed operators and a fixed point theorem for multi-valued maps, we prove an existence result of mild solutions for the control systems under the mixed Lipschitz and Carathéodory conditions. Secondly, we discuss a new set of sufficient conditions for the approximate controllability of the systems. The results are obtained under the assumption that the associated linear system is approximately controllable. Finally, an example is provided to illustrate the proposed theory.

1. Introduction

The concept of controllability has played a central role throughout the history of modern control theory. Moreover, approximate controllable systems are more prevalent and very often approximate controllability is completely adequate in applications; see [28,40]. Therefore, various approximate controllability problems for different kinds of dynamical systems have been investigated in many publications [13,25,29]. The theory of fractional differential equations has received a great deal of attention, and they play an important role in many applied fields, including viscoelasticity, electrochemistry, control, porous media, electromagnetic and so on. Some works have done on the qualitative properties of solutions for these equations; see [1,14,16,24,34] and the references therein. Further, many authors investigated the the approximate controllability of abstract fractional functional differential and integro-differential equations or inclusions in Banach spaces by using fixed point techniques; see [30,35,37,42] and references therein. In addition, impulsive effects exist in many evolution processes in which states are changed abruptly at certain moments of time, involved in such fields as medicine and biology, bioengineering, chemical technology; see [3,7]. Therefore, it seems interesting to study the fractional impulsive differential equations or inclusions [10,15,39,41,43]. For some recent papers on the approximate controllability of these systems in an abstract space; see [6,9,27] and the references therein.

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Stochastic differential equations have attracted great interest because of their practical applications in many areas, such as mechanical, electrical, control engineering and physical sciences and so on; see [31]. Furthermore, besides impulsive effects, stochastic effects likewise exist in real systems. So, impulsive stochastic differential equations describing these dynamical systems subject to both impulse and stochastic changes have attracted considerable attention [4,5,23,36,45]. Recently, there are several papers devoted to the existence, uniqueness and other quantitative and qualitative properties of mild solutions to various semilinear fractional impulsive stochastic differential equations or inclusions (see [8,38,46]). As a result of its widespread use, Zang and Li [48] proved the approximate controllability of fractional impulsive neutral stochastic differential equations with nonlocal conditions. Ahmed [2] established the approximate controllability of impulsive neutral stochastic differential equations with fractional Brownian motion. Zhang et al. [49] concerned with the approximate controllability of nonlinear fractional impulsive stochastic differential equations with state-dependent delay.

However, many impulsive systems arising from realistic models can be described as partial differential equations with not instantaneous impulses. Hernández and O'Regan [20] introduced a new class of abstract impulsive differential equations for which the impulses are not instantaneous. Pierri et al. [33] studied the existence of solutions for a class of semi-linear abstract impulsive differential equations with not instantaneous impulses by using the theory of analytic semigroup. Yu and Wang [47] discussed the existence of mild solutions for periodic boundary value problems with not instantaneous impulse on Banach spaces. Das et al. [12] proved the existence of solution and approximate controllability of a second order neutral partial differential equation with state dependent delay and noninstantaneous impulses. Yan and Lu [44] considered a class of fractional impulsive partial neutral stochastic integro-differential equations with not instantaneous impulses in Hilbert spaces. It is well known that many control systems arising from realistic models can be described as multi-valued fractional impulsive stochastic partial differential or integro-differential equations (see [8,46]). So it is natural to extend the concept of approximate controllability to dynamical systems represented by these impulsive systems. In this paper, we consider the approximate controllability of multi-valued fractional impulsive stochastic partial integro-differential equations with infinite delay and not instantaneous impulses of the form

$$ {}^cD_t^{\alpha}N(x_t) \in AN(x_t) + \int_0^t R(t-s)N(x_s)ds + Bu(t) + F(t, x_t)\frac{dw(t)}{dt}, \quad t \in (s_i, t_{i+1}],\ i = 0, 1, \ldots, N, \tag{1.1} $$

$$ x_0 = \varphi \in \mathcal{B}, \quad x'(0) = 0, \tag{1.2} $$

$$ x(t) = I_i(x_{t_i}) + g_i(t, x_t), \ \ t \in (t_i, s_i],\ i = 1, \ldots, N, \tag{1.3} $$

where the state $x(\cdot)$ takes values in a separable real Hilbert space H with inner product $\langle\cdot,\cdot\rangle_H$ and norm $\|\cdot\|_H$. Let K be another separable Hilbert space with inner product $\langle\cdot,\cdot\rangle_K$ and norm $\|\cdot\|_K$. Suppose $\{w(t) : t \geq 0\}$ is a given K-valued Wiener process with a covariance operator $Q > 0$ defined on a complete probability space $(\Omega, \mathcal{F}, P)$ equipped with a normal filtration $\{\mathcal{F}_t\}_{t\geq 0}$, which is generated by the Wiener process w. Here ${}^cD_t^{\alpha}$ is the Caputo fractional derivative of order $\alpha \in (1, 2)$; A, $(R(t))_{t\geq 0}$ are closed linear operators defined on a common domain which is dense in $(H, \|\cdot\|_H)$, and the control function $u \in L^p_{\mathcal{F}}(J, U)$, a Hilbert space of admissible control functions, $p \geq 2$ be an integer, and B is a bounded linear operator from a Banach space U to $H.D_t^{\alpha}\nu(t)$ represents the Caputo derivative of order $\alpha > 0$ defined by $D_t^{\alpha}\nu(t) = \int_0^t g_{n-\alpha}(t-s)\frac{d^n}{ds^n}\nu(s)ds$, where n is the smallest integer greater than or equal to α and $g_\beta(t) := \frac{t^{\beta-1}}{\Gamma(\beta)}, t > 0, \beta \geq 0$. The time history $x_t : (-\infty, 0] \to H$ given by $x_t(\theta) = x(t+\theta)$ belongs to some abstract phase space $\mathcal{B}$ defined axiomatically. Let $0 = t_0 = s_0 < t_1 \leq s_1 \leq t_2 < \ldots < t_{N-1} \leq s_N \leq t_N \leq t_{N+1} = b$, be prefixed numbers, and $F, G, N(\psi) = \psi(0) - G(t, \psi), \psi \in \mathcal{B}, I_i, g_i (i = 1, \ldots, N)$, are given functions to be specified later. The initial data $\{\varphi(t) : -\infty < t \leq 0\}$ is an $\mathcal{F}_0$-adapted, $\mathcal{B}$-valued random variable independent of the Wiener process w with finite second moment.

The first novelty of this paper is an initial study on the approximate controllability problems for a new class of multi-valued fractional impulsive partial stochastic integro-differential equations with infinite delay and not instantaneous impulses in Hilbert spaces, which is expressed in the form (1.1)–(1.3). In the model, the impulses start abruptly at the points t_i and their action continue on a finite time interval $[t_i, s_i]$. To be precise, the function x takes an abrupt impulse at t_i and follows different rules in the two subintervals $(t_i, s_i]$ and $(s_i, t_{i+1}]$ of the interval $(t_i, t_{i+1}]$. The term $I_i(x_{t_i})$ means that the impulses are also related to the value of $x(t_i) = x(t_i^-)$. This situation as an impulsive action which starts abruptly and stays active on a finite time interval. We note that if $I_i = 0$, then the system reduces to impulsive systems with not instantaneous impulses for the case $x(t) = g_i(t, x_t)$, which extend the many possible models reported in [12,20,33,44,47]. When $t_i = s_i$ for all i, and (1.3) takes the form of $\Delta x(t_i) = I_i(x_{t_i}) = x(t_i^+) - x(t_i^-)$ with $x(t_i^+), x(t_i^-)$ representing the right and left limits of $x(t)$ at $t = t_i$, system (1.1)–(1.3) reduces to impulsive systems with the fixed impulses. With regard to this matter, see [8,38,46,49] and the references therein. The control system (1.1)–(1.3) involve a wide area of applications in physics and mathematics, in this paper we will study the approximate controllability problem, which is natural generalizations of the concept of approximate controllability for fractional impulsive stochastic differential equations well known in the theory of infinite dimensional systems.

The second novelty of this paper is we obtain the approximately controllability for system (1.1)–(1.3) under weaker conditions in the sense of the fractional power arguments. Specifically, we only concerned with the case in which associated operators are analytic and F satisfies the Carathéodory condition and $G, I_i, g_i (i = 1, \ldots, N)$ are Lipschitz continuous. To do this, we discuss this problem by introducing a more appropriate concept for pth moment of mild solutions. Also it establish