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Linear optimal estimation for discrete-time systems with measurement-delay and packet dropping

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ABSTRACT

This paper is concerned with the remote estimation problem for measurement-delay system over a packet-dropping network. The packet-dropping phenomenon in every communication channel is described by an independent and identically distributed Bernoulli process. An optimal linear filter is provided in the linear minimum mean square error sense by applying the reorganized innovation analysis approach. The solution to the filter is given in terms of two Riccati difference equations and one Lyapunov difference equation. Further, the infinite horizon filter is investigated under the condition of system stability. In the end, we supply a numerical example to show the effectiveness of our proposed estimation approach.

1. Introduction

Recently, the estimation problems for networked control systems [1,2] have captured much attention due to the wide applications in communication systems [3,4], image processing [5] and so on. However, communication networks are usually unreliable and may lead to packet losses and delays due to inherent limited bandwidth. The packet losses and delays can degrade the performance or even trigger systems instability. Hence, it is not surprising that the study of the state estimation problem for systems with packet losses and time-delays has exerted a tremendous fascination on a host of scholars [6–13]. In the literature, there have been two approaches for describing the packet loss phenomenon in the network-based systems. The first approach is to use a system with Markovian jumping parameter to denote random packet loss model [14]. The second approach is to formulate the packet loss as an independent Bernoulli process [15–18]. In this paper we are interested in the second approach.

The early work on the Kalman filtering with packet losses can be traced back to [15]. By describing the uncertainty via a sequence of independent and identically distributed (i.i.d.) Bernoulli process, the author derives the estimator by utilizing the statistics of the unobserved Bernoulli process in linear minimum mean square error (LMMSE) sense. The measurement in [16] is either received in full or lost completely. When all the sensor measurements are encoded together and sent over the network in a single packet, the Kalman filter either receives the complete observation if the packet is correctly received or none of the observation if the packet is lost or delayed. Liu and Goldsmith [17] generalize [16] by assuming that the measurements are sent to the filter via two communication channels. The filter in [16,17] is called Kalman filtering with intermittent observations, which exploits additional information concerning the packet arrival indicator sequence by

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time-stamp technique. Zhang et al. [18] propose a new suboptimal estimator under a new performance index, which improves the performance of the LMMSE filter and possesses better properties of stability than the Kalman filtering with intermittent observations. Considering that in practical networked systems, the measurements usually cannot be encapsulated into one data packet and multiple measurements may be transmitted through different communication channels, Gao and Chen [19] extend the results in [18] to a more general case.

Motivated by the preceding works about measurement losses and time-delay systems, we study linear estimation for discrete time systems with measurement delay and packet dropping in this paper. The major contributions of this paper are as follows: (i) the measurements are allowed to be transmitted through different communication channels with packet losses and each measurement loss process is described by an i.i.d. Bernoulli process. Also, the measurements are assumed to possess delay. Our model is more general and practical compared with [15,16,18,19]. A closed-form result is obtained by applying the Hadamard product flexibly. (ii) Based on the reorganized innovation analysis approach, the estimator is derived in terms of two Riccati difference equations of the same dimension as that of the original system and one Lyapunov difference equation. Compared with the classical augmentation approach [20] or partial difference Riccati equation approach [21,22], our approach on estimation is computationally much more efficient.

The organization of this paper is as follows. In Section 2, we present the problem statement, some assumptions and remarks. In Section 3, we provide the main result. By using the approach of reorganized innovation analysis and the projection theory, we obtain the finite horizon filter; then the infinite horizon estimation is considered. A numerical example is given to illustrate the effectiveness of the proposed approach in Section 4. In Section 5, we provide some concluding remarks.

Notation: Throughout this paper, the superscripts "−1" and "T" represent the inverse and transpose of a matrix. $\mathcal{R}^n$ denotes the n-dimensional Euclidean space. I denotes an identity matrix of appropriate dimension. $\mathcal{R}^{n\times m}$ is the set of all $n \times m$ real matrices. $\delta_{ij} = 0$ for $i \neq j$ and $\delta_{ii} = 1$. $\mathcal{L}\left\{\{y(s)\}_{s=0}^{k}\right\}$ denotes the linear subspace spanned by the measurement sequence $\{y(0), \dots, y(k)\}$. The Hadamard product is denoted by $\odot$. Let $diag\{\lambda_1, \dots, \lambda_n\}$ stands for the diagonal matrix having $\lambda_1, \dots, \lambda_n$ as its diagonal elements. Furthermore, the mathematical expectation operator is denoted by E. Matrices, if the dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problem statement

Consider the linear system as follows

$$x(k+1) = Ax(k) + n(k), \tag{1}$$

$$y_0(k) = \xi(k)B_0x(k) + v_0(k), \tag{2}$$

$$y_1(k) = \theta(k)B_1x(k-d) + v_1(k), k \geq d, \tag{3}$$

where $x(k) \in \mathcal{R}^n$, $y_0(k) \in \mathcal{R}^{m_1}$ and $y_1(k) \in \mathcal{R}^{m_2}$ are the state, current measurement and delayed measurement, respectively. $n(k)$, $v_0(k)$ and $v_1(k)$ are white noises with zero mean and covariances $E\{n(k)n^T(j)\} = Q\delta_{k,j}$, $E\{v_0(k)v_0^T(j)\} = R_0\delta_{k,j}$, $E\{v_1(k)v_1^T(j)\} = R_1\delta_{k,j}$ respectively. Here $\xi(k) = diag\{\xi_1(k), \dots, \xi_{m_1}(k)\}$, $\theta(k) = diag\{\theta_1(k), \dots, \theta_{m_2}(k)\}$. The mutually uncorrelated and identically distributed (i.i.d.) Bernoulli random variables $\xi_i(k)$ and the $\theta_i(k)$ are employed to describe, respectively, the packet losses phenomenon in the m_1 channels and m_2 channels with $Pr\{\xi_i(k) = 1\} = \alpha_i$, $Pr\{\xi_i(k) = 0\} = 1 - \alpha_i$, $Pr\{\theta_i(k) = 1\} = \beta_i$ and $Pr\{\theta_i(k) = 0\} = 1 - \beta_i$. The initial state $x(0)$ is a random vector with mean 0 and covariance matrix $D(0)$. The random processes $n(k)$, $v_0(k)$, $v_1(k)$, $\xi(k)$, $\theta(k)$ for all k and the initial state $x(0)$ are mutually independent.

For convenience, the measurement $y(k)$ can be rewritten as the following:

$$y(k) = \begin{cases} y_0(k), & 0 \leq k < d, \\ \begin{bmatrix} y_0(k) \\ y_1(k) \end{bmatrix}, & k \geq d, \end{cases} . \tag{4}$$

Problem. For the given measurements $\{y(k)\}_{k=0}^{N}$, find an LMMSE estimator $\hat{x}(k \mid k)$ of $x(k)$. Further, we consider the infinite horizon LMMSE filter $\hat{x}(k \mid k)$ of $x(k)$.

Remark 2.1. From the distribution of $\xi_i(k)$, it is easy to obtain that $E\{\xi_i(k)\} = \alpha_i$, $E\{(\xi_i(k) - \alpha_i)^2\} = \alpha_i(1 - \alpha_i)$, $E\{\xi_i(k)(1 - \xi_i(k))\} = 0$, $E\{[\xi_i(k) - \alpha_i][\xi_j(l) - \alpha_j]\} = \alpha_i(1 - \alpha_i)\delta_{k,l}\delta_{i,j}$. $E\{\xi_i(k)\xi_j(l)\} = \alpha_i\alpha_j$ for $k \neq l$ or $i \neq j$. For the same line, one has $E\{\eta_i(k)\} = \beta_i$, $E\{(\eta_i(k) - \beta_i)^2\} = \beta_i(1 - \beta_i)$, $E\{\eta_i(k)(1 - \eta_i(k))\} = 0$, $E\{[\eta_i(k) - \beta_i][\eta_j(l) - \beta_j]\} = \beta_i(1 - \beta_i)\delta_{k,l}\delta_{i,j}$. $E\{\eta_i(k)\eta_j(l)\} = \beta_i\beta_j$ for $k \neq l$ or $i \neq j$.

Remark 2.2. In sensor networks, it is convenient to connect a packet with time stamps [16,17,19]. However, in the case of limited bandwidths the packet maybe without time stamps. In this paper, we will solve the LMMSE estimation problem based on the packets without time stamps. Compared with the models in [15,16,18,19], the models (1)–(3) are more general. As for time-delay systems, we can settle the estimation problem by using the state augmentation approach [20] or partial difference Riccati equation approach [21]. However the augmented approach or partial difference Riccati equation approach may bring expensive computational cost when the delay d is large. In the following we will use the reorganized innovation approach to deduce the estimation problem instead of the augmented approach.