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Local convergence of a trust-region algorithm with line search filter technique for nonlinear constrained optimization ☆

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ABSTRACT

A trust-region algorithm in association with line search filter technique for solving nonlinear equality constrained programming is proposed in this paper. In the current iteration, the trial step providing sufficient descent is generated by solving a corresponding trust-region subproblem. Then, the step size is decided by backtracking line search together with filter technique to obtain the next iteration point. The advantage of this method is that resolving trust-region subproblem many times to determine a new iteration point in traditional trust-region method can be avoided and hence the expensive computation can be lessened. And the difficult decisions in regard to the choice of penalty parameters in the merit functions can be avoided by using filter technique. Second order correction steps are introduced in the proposed algorithm to overcome Maratos effect. Convergence analysis shows that fast local convergence can be achieved under some mild assumptions. The preliminary numerical results are reported.

1. Introduction

Nonlinear constrained optimization algorithms are applied in many fields of science and engineering such as chemical engineering, structural engineering, transportation analysis, energy distribution, radar applications, modeling and mechanical design networks, circuit design, and so on [2].

In this paper, we focus on nonlinear equality constrained optimization and construct the effective algorithms. A general nonlinear equality constrained optimization can be written as

$$\underset{x\in\mathbb{R}^n}{\text{minimize}} \quad f(x) \tag{1a}$$

$$\text{subject to} \quad c(x) = 0, \tag{1b}$$

where the objective function $f:\ \mathbb{R}^n \to \mathbb{R}$ and the equality constraints $c:\ \mathbb{R}^n \to \mathbb{R}^m$ with $m < n$ are twice continuously differentiable.

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Filter method was initially introduced by Fletcher and Leyffer in [4] to guarantee the global convergence of the algorithm for solving problem (1). The idea is motivated by the aim of avoiding the difficult decisions in regard to the choice of penalty parameters in the merit functions. Numerical experience with the technique in a sequential quadratic programming (SQP) trust-region algorithm reported in [4] is very encouraging and promising. The global convergence results for the related algorithm were established by Fletcher et al. in [3,5]. Wächter and Biegler proposed line search filter methods for nonlinear programming and showed the global convergence in [19]. The local convergence results were presented in [20]. Other researchers also proposed various filter methods and presented the corresponding convergence analysis (see, e.g., [6,7,13,14,16,17,21,22,24]).

The trust-region method is a well-accepted technique in nonlinear optimization to assure global convergence. However, the search direction generated in trust-region subproblem must satisfy strict interior feasibility which results in computational difficulties. It is possible that the trust-region subproblem with the strict feasibility constraints needs to be resolved many times before obtaining an acceptable step and hence the computational efforts for completing one iteration might be expensive. But line search methods choose its candidate step lengths appropriately by using a so-called backtracking approach (see Algorithm 3.1 in [10]) which is easier to implement and would not cost so much computation. In this circumstance it would be advantageous to perform a backtracking line search. The idea of combining the trust-region and line search backtracking technique suggested in [11] (also, see [1]) motivates to switch to the line search technique by employing the backtracking steps at a trial step which may be unacceptable in the trust-region method, since the trial step should provide a direction with sufficient descent. Zhu proposed affine scaling trust-region methods in association with nonmonotonic interior backtracking line search technique (see [25,26]). These methods can assure global convergence while consume less computational efforts. In the companion paper [12], we have discussed the advantage of combining trust-region method and line search in detail. Furthermore, the comparison of numerical experiments with two traditional trust-region methods [18,23] supported this advantage. The global convergence was also analyzed in [12].

In this paper, we analyze the local convergence properties of the trust-region algorithm proposed in the companion paper [12]. It has been noted by Fletcher and Leyffer that the filter method, similar to a penalty function approach, can suffer from the Maratos effect which can result in poor local convergence (see [4]). To overcome Maratos effect, we introduce second order correction steps. In this paper we show that this modification is indeed able to prevent the Maratos effect.

The remainder of this paper is organized as follows. In Section 2 we revisit the trust-region based filter line search procedure from the companion paper [12] and state the overall algorithm introducing second order correction steps. The local convergence analysis is presented in Section 3. Numerical experiments are reported in Section 4.

Notation: $\|\cdot\|$ and $\|\cdot\|_\infty$ denote the Euclidean norm and l_∞ norm, respectively. The inner product of vectors $x, y \in \mathbb{R}^n$ is denoted by x^Ty. The ith component of a vector $v \in \mathbb{R}^n$ is denoted by $v^{(i)}$. We denote by $O(s_k)$ a sequence $\{v_k\}$ satisfying $\|v_k\| \le \beta\|s_k\|$ for some constant $\beta > 0$ independent of k, and by $o(s_k)$ a sequence $\{v_k\}$ satisfying $\|v_k\| \le \beta_k\|s_k\|$ for some positive sequence $\{\beta_k\}$ with $\lim_{k\to\infty} \beta_k = 0$.

2. Development of the algorithm

In this section, we first describe the components of the algorithm to be proposed and then formally state the overall algorithm in detail.

2.1. Step computation

Given a starting point x_0, the trust-region method combining filter line search technique that we discuss generates a sequence of improved estimates x_k of the solution for problem (1). To this end, at iteration k, we choose a trust-region radius Δ_k and consider a trust-region quadratic programming (QP(x_k, Δ_k)):

$$\underset{d\in\mathbb{R}^n}{\text{minimize}} \quad m_k(x_k+d) \overset{\text{def}}{=} f_k + g_k^T d + \frac{1}{2} d^T B_k d \tag{2a}$$

$$\text{subject to} \quad A_k^T d + c_k = 0, \tag{2b}$$

$$\|d\| \le \Delta_k, \tag{2c}$$

where $f_k = f(x_k), g_k = g(x_k) \overset{\text{def}}{=} \nabla f(x_k), c_k = c(x_k), A_k = A(x_k)$ is the transpose of the Jacobian of the constraint $c(x)$ at x_k, and where the symmetric matrix B_k denotes the Hessian $\nabla^2_{xx}\mathcal{L}(x_k, \lambda_k)$ of the Lagrangian $\mathcal{L}(x, \lambda) \overset{\text{def}}{=} f(x) + \lambda^T c(x)$, or this Hessian's approximation.

Apparently, (QP(x_k, Δ_k)) may be infeasible. We do not try to solve problem (2). At each iteration, a new step d_k is formed as the sum of a normal step n_k and a tangential step t_k, i.e., $d_k = n_k + t_k$.

Assume that A_k has full column rank. We can compute the normal step n_k as following:

$$n_k = -A_k(A_k^T A_k)^{-1} c_k. \tag{3}$$

Furthermore, to obtain sufficient decrease in model function and ensure that sufficient progress is made on satisfying the constraints, we require that

$$\|n_k\| \le \beta_1 \min\{1, \beta_2 \Delta_k^{\beta_3}\}\Delta_k, \tag{4}$$

where $\beta_1 \in (0, 1)$, $\beta_2 \in (0, 1)$ and $\beta_3 > 0$ are fixed constants. If (4) does not hold, the algorithm goes to a *feasibility restoration phase* which is discussed shortly. We first focus on the case that (4) holds.