



Stability and global dynamic of a stage-structured predator–prey model with group defense mechanism of the prey



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ARTICLE INFO

MSC:
58F05
70H99

Keywords:
Predator–prey model
Prey age structure
Defense
Switching

ABSTRACT

In this paper we describe the flow of a predator–prey model where the predator feeds on two age classes of prey. Some basic features of the flow are proved under very mild hypothesis on the functional responses. To study the relationship between the ecological parameters and the behavioral ones, we also analyze a special case of the model where modified versions of the Holling type II functional responses are considered to take into account both a certain interference of the age classes of the prey on the predator activity and a defense mechanism of the juvenile class of the prey. Indeed, we find that independently of the profit that each one of the prey age class provides to the predator, the coexistence of the predator and the prey populations is only possible for a bounded range of the carrying capacity due to the joint action of the defense mechanism and the interference of the prey and the age.

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1. Introduction

The importance of the age class structure in the dynamics of the interacting species has been recognized for a long time. For instance, in [10] and [11], Hastings noticed how the models of predator–prey systems were used to examine questions about stability and have ignored the remarkable role played by the age structure of the prey and the age dependent predation. The role of the prey size is recognized as a factor driving selective predation. Selective predation on profitable prey species can ensure efficient exploitation of resources, [25]. In fact, in the literature we find a lot of natural systems where predation and age class are crucial factors in regulating population growth. For instance, Dörner et al. ([7]) have reported that *Perca fluviatilis* is important in controlling the juvenile fish abundance and thus play an essential role in structuring the fish community; Andersson in [2] concludes that perch selecting the larger individuals can be the cause that produces a population of small, lightly colored isopods observed in shallow lakes. The population data of sparrowhawks and redshanks obtained by Quinn and Cresswell [23], show that the management of prey population may sometimes be best achieved by manipulating prey vulnerability measured by the size-body, rather than by culling their predators. The impact of age-dependent vulnerability of the prey in seasonal predation has been analyzed in [18]. A ratio-dependent functional response was introduced in a predator prey model analyzed in [24], where it was proved that the permanence of the system depends on the mortality rates of the immature class and the predator. This is an interesting fact because only the mature class is preyed. When the selective predation is on the most abundant prey, the

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phenomenon is known as switching and has been considered in many papers, see for example, [14,16,17,19,21,22,26], and in a different context [1,3–5,28]. Another important aspect that could be present in a predator–prey relationship is the ability of the prey to better defend themselves when their number is large. Pairs of musk-oxen can be successfully attacked by wolves but groups often are not attacked [27]. Examples of this kind of group defense can be found in [9,20]. In a context of pest management, a predator–prey model was considered in [15], where the predator only feeds on the reproductive class which presents a group defense; they provide sufficient conditions for the pest extinction. In this paper, we are concerned with the effect of age structure of the prey on the dynamics of a predator–prey interaction when the prey presents a density dependent defense mechanism and the predator tends to feed on the most numerous prey age class (switching effect). In Section 2 we introduce a model specifically constructed to analyze the effect of predation on a reproductive class of the prey, when the non-reproductive class presents a group defense mechanism. It is worth to say, that in contrast with similar models (see [24], for instance), the birth rate is density-dependent. Our model can be useful to study the dynamics of a predator–prey interaction, when both age classes of the prey do not live in the same habitat. In Section 3 we give some general properties of the flow which are valid under very mild conditions of the functional responses. The equilibrium solutions are analyzed in Section 4. Section 5 is devoted to describe the bifurcation process of a system in which we have incorporated a defense mechanism of the younger individuals and both classes can interfere on the predation activity. We find that independently of the ecological scenario considered, assuming both the prey defense and the interference, the stable coexistence is possible only if the carrying capacity is neither too small nor too large. Moreover, in an apparent paradox, a low mortality of the reproductive class can lead to the predators extinction. This feature is a consequence of the group defense of the prey as we have explained in Section 6.

2. Predator–prey model with a two age-classes of the prey

To study the impact of the age-dependent vulnerability of the prey we consider the following model

$$\begin{aligned}\dot{x} &= r\left(1 - \frac{x}{K}\right)y - \nu x - f(x, y)xz = F(x, y, z), \\ \dot{y} &= \nu x - \mu y - g(x, y)yz = G(x, y, z), \\ \dot{z} &= (\gamma x f(x, y) + \lambda y g(x, y) - D)z = zH(x, y, z).\end{aligned}\tag{2.1}$$

The term x stands for the population of the juvenile prey, y is the reproductive class of the prey and z represents the predator population. The constant K is the carrying capacity of the habitat where the juvenile class evolves. The impact of the prey density on the intrinsic reproductive rate r is given by the factor $(1 - \frac{x}{K})$ considered in the first equation of the system (2.1). Juvenile individuals become adult organisms with a rate ν . The functions f and g are the functional responses of the juvenile and the reproductive age class, respectively. In this work the functional responses f and g are given by

$$f(x, y) = \frac{1}{1 + x^2 + y} \quad \text{and} \quad g(x, y) = \frac{1}{1 + x + y},$$

so the juvenile population presents a density dependent defense mechanism and incorporating the density y in the predation rate on the juvenile class we simulate an interfering effect of the adult class on the predation over the juvenile class. The same meaning has the presence of the juvenile density in the predation rate on the adult class. Our aim is to study the way that defense and interference affect the relationship between the predation parameters and the environment's capacity. In this model, unlike the similar model studied by Falconi in [8], the juvenile class of the prey can grow in a patch different from that where the adult population develops its activity. We also remark that the results of Section 3 are still valid for arbitrary functions f and g , assuming the following conditions:

1. F1: $f(x, y) > 0, g(x, y) > 0$ are continuously differentiable functions defined in \mathbb{R}_+^2 ,
2. F2: $xf(x, y), yg(x, y)$ are bounded functions in \mathbb{R}_+^2 . $\lim_{x \rightarrow 0} xf(x, y) = 0$, and $\lim_{y \rightarrow 0} yg(x, y) = 0$.

3. General properties of the flow

As usual, we are concerned with the properties of the flow for $t \geq 0$, in the first octant of \mathbb{R}^3 given by $\mathbb{R}_+^3 = \{(x \geq 0, y \geq 0, z \geq 0)\}$. Let $\varphi(t) = (x(t), y(t), z(t))$ be a solution of the system (2.1) defined on its maximal domain of definition $(0, \omega_+)$. Note that the octant \mathbb{R}_+^3 is positively invariant.

Proposition 1. *The octant \mathbb{R}_+^3 is invariant for $t \geq 0$, under the flow defined by the system (2.1).*

Proof. It is enough to observe the behavior of the vector field associated with the system (2.1). In fact, on the plane $x = 0$ we have $\dot{x} > 0$, $\dot{y} < 0$, then the vector field points towards the interior of \mathbb{R}_+^3 ; analogously, on the plane $y = 0$ the vector field points towards the interior of \mathbb{R}_+^3 , since we have that $\dot{x} < 0$ and $\dot{y} > 0$; finally, the plane $z = 0$ is invariant, since $\dot{z} = 0$. \square

Now, we prove that all the trajectories of the flow are bounded.

Proposition 2. *Let $\varphi(t)$ be a non equilibrium solution of the system (2.1). Then $x(t), y(t)$ and $z(t)$ are bounded on its maximal domain of definition.*

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