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Finite-time consensus for second-order stochastic multi-agent systems with nonlinear dynamics

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ABSTRACT

In this study, the problem of finite-time consensus in probability for second-order stochastic multi-agent systems with non-identical nonlinear dynamics is investigated. Based on the stochastic finite-time stability theorems and adding a power integrator technique, we propose the distributed control algorithm for the stochastic multi-agent systems, which can guarantee all agents converge to consensus in finite time with probability one under an undirected graph. Two simulation examples are presented to demonstrate the effectiveness of the proposed methods.

1. Introduction

In recent years, much attention has been paid to distributed cooperative control of multi-agent systems due to its broad applications in unmanned air vehicles, flocking, sensor networks, multiple autonomous robots and so forth [1–4]. In cooperative control, one of the most important issues is to design distributed protocols based on local relative information such that all agents converge to consensus. The consensus means the states of all the agents converge to a common value under some control schemes for each agent. Many consensus algorithms have been investigated in [5–14].

Note that most of the consensus algorithms in [6,8,9,12] are asymptotically convergent algorithms with infinite settling time, which means the consensus cannot be achieved in finite time. In fact, the finite-time consensus algorithms are more desirable, since the finite-time control not only provides a faster convergence rate but also guarantees the closed-loop systems have better disturbance rejection ability [16–21]. Recently, many finite-time consensus algorithms have been proposed for first-order and second-order multi-agent system [14,22–27]. For example, the finite time consensus distributed protocols for the leaderless and leader-follower second-order multi-agent systems were designed in [22] by using the adding a power integrator technique; the finite time containment control was solved in [25] without velocity measurements based on homogeneous method; the finite time attitude consensus control for multiples spacecraft formation flying was solved in [27] based on terminal sliding mode method; the adaptive finite-time leaderless consensus control of multi-agent systems consisting of a group of nonlinear mechanical systems with parametric uncertainties was solved in [14] based on the adding a power integrator technique combined with adaptive method. All these references, however, only focus on the deterministic systems.

In practice, the evolution of dynamical systems is always affected by various random disturbances and uncertainties such as noise from the unpredictable environmental conditions [35,36]. For example, the random disturbances are considered in some practical systems, including the neural network systems [37], Lagrangian systems [38], spacecraft attitude systems [39]. Recently, the mean-square consensus controls of discrete and continuous multi-agent systems with stochastic communication noises were

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solved in [40,41], respectively; the stochastic consensus problem of first-order multi-agent systems with nonlinear dynamics and state-dependent noise perturbation was discussed in [42]. However, the systems can only achieve asymptotic consensus under the algorithms in [40–42]. The finite-time convergence performance of stochastic systems was more absorbing [30,31], and the finite-time controls for stochastic systems were studied in [32,33,45–47], where the adding a power integrator technique was an effective method to solve the finite-time stabilization of stochastic nonlinear system [32,33]. It should be pointed out that the adding a power integrator technique has not been investigated for the distributed control of stochastic multi-agent systems in the existing literatures. Moreover, the finite-time consensus problem for first-order multi-agent systems with random disturbances and single integrator dynamic was studied [43,44], but the problem of finite-time consensus for second-order stochastic multi-agent systems has not been investigated to date, especially for multi-agent systems with nonlinear dynamics, which are often encountered by external causes [9,15].

Based on the above discussions and motivated by the new results on finite-time stability in probability theorems for stochastic systems in [32–34], this paper will devote to study the finite-time consensus in probability problem for stochastic second-order multi-agent systems in undirected networks. The contributions are stated as follows: (a) The adding a power integrator technique is extended to solve the finite-time consensus of stochastic multi-agent system with non-identical nonlinear dynamics; (b) The finite-time stability in probability is proved directly by constructing a proper Lyapunov function and the explicit expression of the stochastic settling time is obtained; (c) The proposed method can also be used to design distributed exponential consensus laws for second-order multi-agent systems with random disturbances.

The rest of the paper is organized as follows. Some preliminaries are given in Section 2. The main results are presented in Section 3. Three numerical examples are given in Section 4 to show the effectiveness of the proposed design methods. Conclusions are drawn in Section 5.

Throughout this paper, $\Re^n$ and $\Re^{n\times m}$ denote the n-dimensional and $n \times m$-dimensional Euclidean spaces, respectively; I denotes the identity matrix with appropriate dimensions; the superscripts T stands for matrix transposition; $\lambda_{\min}(X)$ denotes the minimum eigenvalue of matrix X; $|\cdot|$ refers to the absolute value, and $\|\cdot\|$ refers to the Euclidean vector norm or the induced matrix 2-norm; $\otimes$ denotes the Kronecker product; $\mathrm{sig}^r(\cdot) = \mathrm{sgn}(\cdot)|\cdot|^r$; $\mathbf{1} = [1, \ldots, 1]^T$ and $\mathbf{0} = [0, \ldots, 0]^T$; $a^b = [a_1^b, \ldots, a_n^b]^T$, where $a = [a_1, \ldots, a_n]^T \in \Re^n$, $b \in \Re$.

2. Problem formulation

2.1. Stochastic finite-time stability

This subsection reviews the definitions and theorems of finite-time stability in probability for stochastic nonlinear systems given in [28–34]. Consider the stochastic nonlinear system as follows

$$\mathrm{d}x(t) = f(t, x(t))\mathrm{d}t + g(t, x(t))\mathrm{d}w(t), \quad x(0) = x_0 \in \Re^n, \quad t \geq 0 \tag{1}$$

where $x \in \Re^n$ is the state vector, $w(t)$ is an m-dimensional standard Wiener process defined on a complete probability space (Ω, F, P) with the augmented filtration $\{F_t\}_{t\geq 0}$ generated by $\{w_t\}_{t\geq 0}$ and the coefficients $f : \Re_+ \times \Re^n \to \Re^n$ and $g : \Re_+ \times \Re^n \to \Re^{n\times m}$ are continuous in x and t satisfying $f(t, 0) = 0$ and $g(t, 0) = 0$ for all $t \geq 0$.

Definition 1 ([33]). (i) The trivial solution of (1) is said to be finite-time attractive in probability, if the stochastic system admits a solution (either in the strong sense or in the weak sense) for any initial data $x_0 \in \Re^n$, denoted by $x(t, x_0)$; moreover, for every initial value $x_0 \in \Re^n\backslash\{0\}$, the first hitting time $\tau_{x_0} = \inf\{t : x(t, x_0) = 0\} = \inf\{t : |x(t, x_0)| = 0\}$ called stochastic settling time, is finite almost surely, that is, $P\{\tau_{x_0} < \infty\} = 1$. (ii) The trivial solution is said to be stable in probability if for every pair of $\varepsilon \in (0, 1)$ and $r > 0$, there exists $\delta = \delta(\varepsilon, r) > 0$ such that $P\{|x(t, x_0)| < r \text{ for all } t \geq 0\} \geq 1 - \varepsilon$ whenever $|x_0| < \delta$. The trivial solution of (1) is said to be finite-time stable in probability if it is both finite-time attractive and stable in probability.

For a C^2 Lyapunov function V, the differential operator of V with respect to (1) is defined by [32,33]

$$\mathscr{L}V(x) = \frac{\partial V(x)}{\partial x} f(t, x) + \frac{1}{2}\mathrm{trace}\left\{g^T(t, x)\frac{\partial^2 V(x)}{\partial x^2} g(t, x)\right\}. \tag{2}$$

Lemma 1 ([33]). *Assume that there exists a nonnegative radially unbounded function $V \in C^2$, and (1) is autonomous, if the differential operator of V with respect to (1) satisfies $\mathscr{L}V(x) \leq 0$, $\forall x \in \Re^n$, then (1) has a solution for any initial data.*

Lemma 2 ([33,34]). *Assume that (1) admits a solution for each initial vector, if there exists a $C_2^0(\Re^n)$ function $V : \Re^n \to \Re_+$, $\mathscr{K}_\infty$ class functions μ_1 and μ_2, real numbers $c > 0$ and $0 \leq \gamma < 1$, such that for all $t \geq 0$*

$$\mu_1(x) \leq V(x) \leq \mu_2(x), \quad x \in \Re^n \tag{3}$$

$$\mathscr{L}V(x) \leq -cV(x)^\gamma, \quad x \in \Re^n \setminus \{0\} \tag{4}$$

then the origin of (1) is finite-time stable in probability and the stochastic setting time satisfies $E\{\tau_{x_0}\} \leq \frac{V(x_0)^{1-\gamma}}{c(1-\gamma)}$.

The following lemmas will be used in the sequel.