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Periodic solution of a prey–predator model with nonlinear state feedback control

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ABSTRACT

Assume that when the number of pests reaches the certain threshold, pest management strategy will be taken to control pests. Based on this assumption, in this paper, we propose a pest management model with nonlinear state feedback control. We then analyze the dynamic behavior of the model. More precisely, we first investigate the singularity of the model by using method of qualitative analysis; secondly the existence of periodic solution of the model is studied by using successor functions and Poincaré–Bendixson theorem; and then it is followed by the study of the stability of periodic solution; finally, an example with numerical simulations is given to illustrate our conclusions.

1. Introduction and model formulation

Threat of pests to agricultural productions is a serious problem across the world, which in recent years attracts a great attention on the development of effective pest controlling methods. For example, the use of efficient pesticides has greatly improved the efficiency of pest management, but due to the misuse of them, it results in many problems such as pest resistance [1,2], pesticide residues, pest resurgence [3,4], etc.

Integrated pest management (IPM) utilizes a combination of agricultural, biological, chemical, physical, and cultural methods to control pests. Its goal is not to eradicate pests, instead to control the number of the pests under an economic threshold (ET), which is different from traditional ones. As seen IPM not only can control pests effectively, but also protect ecosystems in maximum extent. Therefore, as a long-term controlling methods of pests, farmers welcomed IPM, which have already attracted many scholars' attention [5–11]. However, IPM may cause biological system population change radically due to the variety of manual intervention such as spraying pesticide, releasing natural enemies, trapping and so on. Impulsive differential equations (IDEs) in mathematics is a powerful tool to describe this change. Based on the theory of IDEs [12–17], lots of models with periodic impulsive manual intervention have been developed to study the IPM [18–26,29,30].

However, in the actual pest management, the state-dependent impulsive intervention seems more reasonable and is introduced. For example, Tang [27,28] established the following pest management system with one state-dependent pulse:

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$$\begin{cases} \left.\begin{aligned} \dot{x}(t) &= x(t)(a - by(t)), \\ \dot{y}(t) &= y(t)(cx(t) - d), \end{aligned}\right\} x \neq \mathrm{ET}, \\ \left.\begin{aligned} \Delta x(t) &= -px(t), \\ \Delta y(t) &= \tau, \end{aligned}\right\} x = \mathrm{ET}. \end{cases} \tag{1}$$

In system (1), it is assumed that killing rate of pests p is a constant (proportional to the number of pests) after each spraying insecticides, where $p \in [0, 1)$. However, due to limited resources and pest resistance to pesticides, insecticides show saturation effects killing rate of pests. Then Wang and Tang [31] proposed a new system with nonlinear impulsive effect:

$$\begin{cases} \left.\begin{aligned} \dot{x}(t) &= x(t)(a - by(t)), \\ \dot{y}(t) &= y(t)(cx(t) - d), \end{aligned}\right\} x \neq \mathrm{ET}, \\ \left.\begin{aligned} \Delta x(t) &= -p(x(t))x(t), \\ \Delta y(t) &= \tau, \end{aligned}\right\} x = \mathrm{ET}, \end{cases} \tag{2}$$

in which in order to portray these effects, killing rate takes the form of

$$p(x(t)) = \frac{P_{\max} x(t)}{x(t) + \theta},$$

where $P_{\max} \in [0, 1)$ denotes the maximal killing proportion, θ is the half saturation constant. Motivated by the previous works [27,28,31], we propose the following system with logistic growth and nonlinear state-dependent pulse.

$$\begin{cases} \left.\begin{aligned} \dot{x}(t) &= rx(t)\left(1 - \frac{x(t)}{K}\right) - bx(t)y(t), \\ \dot{y}(t) &= y(t)(cx(t) - d), \end{aligned}\right\} x \neq h, \\ \left.\begin{aligned} \Delta x(t) &= -p(x(t))x(t), \\ \Delta y(t) &= -qy(t) + \alpha, \end{aligned}\right\} x = h, \end{cases} \tag{3}$$

where $\Delta x(t) = x(t^+) - x(t)$, $\Delta y(t) = y(t^+) - y(t)$, r is the intrinsic growth rate of the prey in the absence of the predator. K is usually referred to the environment carrying capacity of saturation level. h is the critical value of economic. q is the death rate of natural enemies due to pesticide. α is the quantity of natural enemies released once and other parameters can be seen in [28]. To be biologically meaningful, we restrict our study in the region of $R^2_+ = \{(x, y)|x \geq 0, y \geq 0\}$.

In systems (1) and (2), authors have discussed the existence of the order-one periodic solutions by using Lambert W function. In system (3), we shall discuss the existence of the order-one periodic solutions by using geometrical method (successor function) which is different from the ones in [27,28,31].

The rest of the paper is organized as follows. In Section 2, we briefly introduce some concepts and fundamental results, which are necessary for future discussions. In Section 3, we will focus our attention on the qualitative analysis of system (3) by neglecting impulsive effect. In Section 4, we will investigate the periodic behavior of solutions of system (3) by considering impulsive state feedback control. Then we show an example and carry out numerical simulations in Section 5, from which it can be seen that all simulations agree with the theoretical results. We finally conclude our paper in Section 6.

2. Preliminaries

Definition 2.1. [17] Consider

$$\begin{cases} \left.\begin{aligned} \dot{x}(t) &= F(x, y), \\ \dot{y}(t) &= G(x, y), \end{aligned}\right\} \text{ for } (x, y) \notin M\{x, y\}, \\ \left.\begin{aligned} \triangle x &= \alpha(x, y), \\ \triangle y &= \beta(x, y), \end{aligned}\right\} \text{ for } (x, y) \in M\{x, y\}. \end{cases} \tag{4}$$

We call M the impulsive set and a continuous mapping $I : I(M) = N$ the impulse function, here N is referred as the phase set. Furthermore, a semicontinuous system is a set defined by the solution maps of system (4), and denoted as $(\mho, f, I, M)$.

Given initial mapping point $P \notin M$. Then we have the following definition.

Definition 2.2. [17] Let $P(P_x, P_y) \in N$ be the initial mapping point, $f(P, T) = P_1((P_{1x}, P_{1y})) \in M\{x, y\}$, $I(P_1) = I(f(P, T)) = P^+((P_x^+, P_y^+)) \in N$. The coordinates of point P and P^+ are denoted as (P_x, P_y) and (P_x^+, P_y^+) respectively, (see Fig. 1). Then the successor function of the point P is defined by $F(P) = F(P^+) - F(P) = P_y^+ - P_y$.

Definition 2.3. [17] If there exists a point $P \in N$ such that $f(P, T) = Q \in M\{x, y\}$ and $I(Q) = I(f(P, T)) = P \in N$, then $f(P, T)$ is called an order one periodic solution of system (4).

Lemma 2.1. *[17] Let (X, Π) be a continuous dynamic system, for two points P_1, P_2, in the pulse phase, if $F(P_1)F(P_2) < 0$, then there exists a point P between P_1 and P_2 such that $F(P) = 0$, that is, the system has order one periodic solution, where F is the successor function.*