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A coalitional value for games on convex geometries with a coalition structure

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ABSTRACT

With respect to games on convex geometries with a coalition structure, a coalitional value named the generalized symmetric coalitional Banzhaf value is defined, which can be seen as an extension of the symmetric coalitional Banzhaf value given by Alonso-Meijide and Fiestras-Janeiro and the Shapley value for games on convex geometries introduced by Bilbao. Based on the established axiomatic system, the existence and uniqueness of the given coalitional value is shown. Meanwhile, a special case is briefly studied.

1. Introduction

Game theory is a powerful tool to research cooperative and competitive relations between players. Since von Neumann and Morgenstern created game theory in 1944, researchers realized that this theory can be applied in many fields such as politics, economy and military. Evolutionary game theory, which aims to understand the emergence of cooperation, is a very hot field currently. For instance, Wang et al. [1] studied evolutionary games on interdependent networks; Wang et al. [2] researched the optimal intermediate level of interdependence the networks; the strategy evolution in the prisoner's dilemma game is studied by Wang et al. [3], Wang and Perc [4], and Perc and Wang [5] that are based on different point of views. Szolnoki et al. [6] showed that the change of the strategy depends not only on the individual, but also on the others. Furthermore, the authors concluded that plugging into the "wisdom of groups" can promote cooperative levels. The evolution of cooperation in multilayer networks is discussed in [7,8]. Based on a utility function, Wang et al. [9] studied the evolution of public cooperation on two interdependent networks. Deng et al. [10] researched belief-based evolutionarily stable strategy (ESS) in terms of Dempster–Shafer evidence theory, which is an extension of the mixed strategy and mixed ESS. Perc and Grigolini [11] analyzed the current researches of evolutionary games and pointed out the future research aspects. Perc et al. [12] reviewed recent advances in the study of evolutionary dynamics of group interactions of spatial games. Wang et al. [13] discussed the coevolution of strategy and network interdependence in the prisoner's dilemma game and showed that the interdependence between networks self-organizes can yield optimal conditions for the evolution of cooperation, whilst Perc and Szolnoki [14] presented a review of recent works on coevolutionary games and outlined directions for future research.

Different to evolutionary game theory that focuses on the evolution of cooperation, cooperative game theory mainly discusses the cooperative mode and the players' payoffs. In traditional cooperative game theory, we usually assume that the players can cooperate freely and get payoffs from the coalitions including him/her. However, to obtain more payoffs and win the competitive

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advantage, there are many cooperative situations where players are joined in coalitions that form a partition or coalitional structure of the set of players. Aumann and Drèze [15] first researched games with a coalition structure and extended the Shapley value to this new framework in such a manner that the coalitions are independent with each other. Owen [16] further studied games with a coalition structure where the probability of cooperation among coalitions is considered and provided the Owen value, whilst Owen [17] introduced the Banzhaf–Owen value for games with a coalition structure, which is an extension of the Banzhaf value for traditional games. Recently, Alonso-Meijde and Fiestras-Janeiro [18] pointed out the Banzhaf–Owen value dissatisfy symmetry in quotient games and gave another solution concept for games with a coalition structure, which is named the symmetric coalitional Banzhaf value. Two axiomatic systems for the symmetric coalitional Banzhaf value are researched by using coalitional total power property. The axiomatic systems for the Owen value are researched by Albizuri [19], Hamiache [20] and Khmelnitskaya and Yanovskaya [21], whilst the characterizations of the Banzhaf–Owen value are discussed by Amer et al. [22] and Alonso-Meijide et al. [23]. Furthermore, multi-choice games with a coalition structure are considered in [24–26].

On the other hand, there is another important kind of games that not all coalitions can be formed. People called it games on precedence constraints. Myerson [27] introduced games with communication situations and defined the generalized Shapley value for the proposed games. Faige and Kern [28] discussed a special kind of the cooperative games under precedence constraints. Gilles and Owen [29] studied games with disjunctive permission structures, and Gilles et al. [30] discussed games with conjunctive permission structures. Bilbao [31] defined games on convex geometries and researched the Shapley value on them. Later, Bilbao et al. [32,33] researched another special kind of games under precedence constraints named games on matroids and studied the Shapley values of two cases. Algaba et al. [34] presented games on antimatroids and researched the Shapley value for this class of games. Bilbao [35] introduced games on augmenting systems and discussed the relationship between augmenting system, antimatroid and convex geometry.

Based on the researches about games with a coalition structure and games on convex geometries, Meng and Zhang [36] researched games on convex geometries with a coalition structure, which can be seen as an extension of games with a coalition structure. Furthermore, the authors studied the generalized Owen value for this kind of games. Based on the symmetric coalitional Banzhaf value [18] for games with a coalition structure, the Shapley value [37] and the Banzhaf value [38] for games on convex geometries, we here research another importance payoff index of games on convex geometries with a coalition structure called the generalized symmetric coalitional Banzhaf value. The existence and uniqueness of this coalitional value are proved by the established axiomatic system. It is worth noticing that this coalitional value can be seen as an extension of several payoff indices.

2. Two kinds of games

2.1. Cooperative games with a coalition structure

Let $P(N)$ be the power set of N, where $N = \{1, 2, \ldots, n\}$ is the finite collection of players. The coalitions in $P(N)$ are denoted by $S, T, \ldots$. The cardinality of any $S \in P(N)$ is denoted by the corresponding lower case s.

A coalition structure $\Gamma = \{B_1, B_2, \ldots, B_m\}$ on player set N is a partition of N, *i.e.*, $\cup_{1\le h\le m} B_h = N$ and $B_h \cap B_l = \emptyset$ for all $h, l \in M = \{1, 2, \ldots, m\}$ such that $h \neq l$, which is denoted by (N, Γ). Let $L(N, \Gamma)$ be the set of all coalitions in (N, Γ). A function $v : L(N, \Gamma) \to \Re_+$, such that $v(\emptyset) = 0$, is called a set function. The set of all set functions in $L(N, \Gamma)$ is denoted by $G(N, \Gamma)$.

Definition 2.1 ([16]). Let $v \in G(N, \Gamma)$, if we have $v^B(R) = v(\cup_{r\in R} B_r)$ for any $R \subseteq M$, then v^B is said to be a quotient game on (N, Γ), where Γ and M as shown above, denoted by (M, v^B).

Let f be a solution on $G(N, \Gamma)$, consider the following properties introduced by Alonso-Meijde and Fiestras-Janeiro [18].

Additivity Let $v, w \in G(N, \Gamma)$. If $v + w \in G(N, \Gamma)$, then $f(N, v + w, \Gamma) = f(N, v, \Gamma) + f(N, w, \Gamma)$.

Null player property Let $v \in G(N, \Gamma)$ and $i \in N$, if $v(S \cup i) = v(S)$ for any $S \in L(N, \Gamma)$ with $i \notin S$, then $f_i(N, v, \Gamma) = 0$.

Symmetry in the unions Let $v \in G(N, \Gamma)$ and $i, j \in B_k \in \Gamma$, if $v(S \cup i) = v(S \cup j)$ for any $S \in L(N, \Gamma)$ with $i, j \notin S$, then $f_i(N, v, \Gamma) = f_j(N, v, \Gamma)$.

Symmetry in the quotient games Let $v \in G(N, \Gamma)$ and $k, l \in M$, if $v^B(R \cup k) = v^B(R \cup l)$ for any $R \subseteq M\backslash\{k, l\}$, then $\sum_{i\in B_k} f_i(N, v, \Gamma) = \sum_{j\in B_l} f_j(N, v, \Gamma)$.

Coalitional total power Let $v \in G(N, \Gamma)$ and $B_k \in \Gamma$, then $\sum\limits_{i\in N} f_i(N, v, \Gamma) = \frac{1}{2^{m-1}} \sum\limits_{k\in M} \sum\limits_{R\subseteq M\backslash k} (v^B(R \cup k) - v^B(R))$.

Remark 2.1. All above properties are very intuitive. Additivity means that the payoff of the sum of two cooperative games is equal to the sum of the cooperative games' payoff; Null player property means that a player is null if his/her contribution to every coalition is zero; Symmetry in the unions means that two player in a union have the same payoff if their contributions to every coalition are equal; Symmetry in the quotient games means that two unions have the same payoff if their contributions to every coalition formed by unions are equal; Coalitional total power means that the payoffs of all players equal to the sum of the players' Banzhaf values.