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Application of a combination production function model

Maolin Cheng [a,*], Minyin Xiang [b]

[a] School of Mathematics and Physics, Suzhou University of Science and Technology, Suzhou 215009, China
[b] School of Mathematics and Statistics, Huangshan University, Huangshan 245041, China

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ABSTRACT

In the analysis of economic growth factors, people often use the production function to analyze the contribution rate of the influencing factors of economic growth. The commonly-used production functions include C–D production function, CES production function and VES production function; the analysis result of three production functions always differ a lot; in order to reduce the difference and increase the reliability of the analysis, this paper combines with three production function models, utilizes the principal component regression, and obtains the composite production function model. Based on the composite production function model, a method which is used to calculate the contribution rate of the economic growth factors is proposed in this paper; based on this method, the deficiencies of Solow's calculation formula are overcome, and the precision estimation is improved. Finally, according to the given method in this paper, the empirical analysis is made on the contribution rate of the economic growth factors for the tertiary industry in China.

1. Introduction

Under some circumstances, the study on economic growth theory can be transformed to the study on production function. During the material production process, production function reflects the mathematical expression of the still existed relation between the combination of production factor input and the actual output. Economists have proposed various production functions, and an influential one was the production function established by the Nobel Economics Prize winner—Professor Solow form U.S. Massachusetts Institute of Technology in 1956

$$Y = AF(K, L),$$

where Y refers to the output, K refers to capital input, L refers to labor input, and A refers to technical level. The model has three main forms [1–5], i.e., the C–D production function, CES production function and VES production function.

The form of C–D (Cobb–Douglas) production function is

$$Y = A_1 e^{\lambda_1 t} K^{\alpha} L^{\beta},$$

where Y refers to the output, K refers to capital input, L refers to labor input, α, β refer to the output elasticity of capital and labor respectively, λ_1 refers to the speed of technical progress, and $A_1, \alpha, \beta, \lambda_1$ are parameters to be estimated.

* Corresponding author.
E-mail addresses: ustscml@163.com (M. Cheng), hsxmy@hsu.edu.cn (M. Xiang).

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The CES (Constant Elasticity of Substitution) production function has various forms, and generally, the following form is adopted

$$Y = A(\delta_K K^{-\rho} + \delta_L L^{-\rho})^{-\frac{\mu}{\rho}}.$$

In this paper, it is revised to

$$Y = A_2 e^{\lambda_2 t}(\delta_K K^{-\rho} + \delta_L L^{-\rho})^{-\frac{\mu_1}{\rho}},$$

where δ_K and δ_L respectively represent the intensive degrees of capital and labor from the perspective of technology; μ_1 represents the homogeneous order or returns to scale rate of the function, $\mu_1 > 0$; ρ is the substitution parameter, $\rho \geqslant -1$; λ_2 refers to the speed of technical progress, and $A_2, \delta_K, \delta_L, \rho, \mu_1, \lambda_2$ are all parameters to be estimated.

The VES (Variable Elasticity of Substitution) production function also has various forms, and the well-known forms include the model proposed by Sato and Hoffman in 1968 and the model proposed by Revankar in 1971.

Revankar's VES production function is

$$Y = AK^{\mu\frac{1}{(1+c)}}\left(L + \frac{b}{1+c}K\right)^{\mu\frac{c}{(1+c)}}.$$

In this paper, it is revised to

$$Y = A_3 e^{\lambda_3 t} K^{\mu_2\frac{1}{(1+c)}}\left(L + \frac{b}{1+c}K\right)^{\mu_2\frac{c}{(1+c)}},$$

where μ_2 represents the returns to scale rate; c is the factor-production elastic parameter; λ_3 refers to the speed of technical progress; $A_3, \mu_2, c, b, \lambda_3$ are parameters to be estimated. During analysis of the economic growth factor, generally one production function is chosen to estimate the contribution rate of economic growth factors, and different models have different estimated results. In order to reduce the different and increase the reliability of result, this paper combines these three production functions. Considering the results of the three production functions have multicollinearity, this paper uses principal component regression [6–11] to obtain the combined production function model. By using the combined production function model, this paper provides a method to calculate the contribution rate of economic growth factors [12–16], and this method is based on that economic variables generally present continuous change of gray model path (it refers to the change of exponential form), overcomes the shortage of Solow's measurement and calculation formula which uses difference to replace differential and increases the measurement and calculation accuracy. At last, the method provided by this paper conducts empirical analysis of contribution rate of economic growth factors of China's tertiary industry.

2. Combined production function model

Among the three production functions, it is difficult to transform the CES and VES production functions to a linear model, its parameter estimation cannot directly use the linear least square method, and here we use the nonlinear regression method. Considering the nonlinear production functional equation

$$Y = f(K, L, \varphi_1, \varphi_2, \ldots, \varphi_p) + \varepsilon,$$

where f is the nonlinear production function, which is a theoretical model; K, L are independent variables; Y is a dependent variable, which is a actual observed value; $\varphi_1, \varphi_2, \ldots, \varphi_p$ are p parameters. The standard for estimated parameters $\varphi_1, \varphi_2, \ldots, \varphi_p$ is similar to linear regression, which is the minimization of error sum of squares. Let

$$\sum_{i=1}^{n}\varepsilon_i^2 = g(\varphi) = \sum_{t=1}^{n}[Y_t - f(K_t, L_t, \varphi_1, \varphi_2, \ldots, \varphi_p)]^2,$$

has a smallest value, and the unknown parameter is obtained through nonlinear least square estimation [17–20]. However, the calculation method for nonlinear estimation is difficult to realize, which usually uses software, and this paper uses the MATLAB software to estimate the parameter.

For the C–D production function, the theoretical model is

$$Y_1 = A_1 e^{\lambda_1 t} K^{\alpha} L^{\beta},$$

through the nonlinear least square method, we solve the parameters $A_1, \alpha, \beta, \lambda_1$.

For the CES production function, the theoretical model is

$$Y_2 = A_2 e^{\lambda_2 t}(\delta_K K^{-\rho} + \delta_L L^{-\rho})^{-\frac{\mu_1}{\rho}},$$

through the nonlinear least square method, we solve the parameters $A_2, \delta_K, \delta_L, \rho, \mu_1, \lambda_2$.