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Almost periodic solutions of a discrete almost periodic logistic equation with delay

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ABSTRACT

In this paper, we consider an almost periodic discrete logistic equation with delay. By constructing suitable Lyapunov functional and almost periodic functional hull theory, a sufficient condition is obtained for the existence of a unique almost periodic solution which is globally attractive. An example together with its numerical simulation shows the feasibility of our main result.

1. Introduction

In this paper, we study the following discrete logistic equation,

$$x(n+1) = x(n)\exp\left(r(n)\left(1 - \frac{x(n-\tau)}{K(n)}\right)\right), \tag{1.1}$$

where $\{r(n)\}$ and $\{K(n)\}$ are bounded nonnegative almost periodic sequences (see Section 2 for the definition) such that

$$0 < r^L \leqslant r(n) \leqslant r^M, \quad 0 < K^L \leqslant K(n) \leqslant K^M, \quad n \in Z.$$

Here, denote the sets of integers and nonnegative integers by Z and Z^+ , respectively. For any bounded sequence $\{g(n)\}$ defined on Z, denote $g^M = \sup_{n\in Z} g(n)$ and $g^L = \inf_{n\in Z} g(n)$.

We consider system (1.1) with the following initial conditions:

$$x(\theta) = \varphi(\theta) > 0, \quad \theta \in N[-\tau, 0] = \{-\tau, -\tau+1, \ldots, 0\}.$$

It is not difficult to see that solutions of (1.1) are well defined for all $n \geqslant 0$ and satisfy $x(n) > 0$.

Though lots have been done for population models described by differential equations (see [1–8]), it has been found that the dynamics of their discrete analogues is rather complex and much richer (see [9–16]). Fox example, Li and Zhang [17] considered an almost periodic discrete delay logistic equation with feedback control, and investigated the permanence and existence of a unique globally attractive positive almost periodic sequence solution of the system. Furthermore, Li, Zhang and Ye [18] discussed an almost periodic discrete predator–prey models with time delays. Sufficient conditions for the permanence of the system and the existence of a unique uniformly asymptotically stable positive almost periodic sequence solution were obtained.

Zhou and Zou [19] considered the following discrete periodic logistic equation

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$$x(n+1) = x(n)\exp\left(r(n)\left(1 - \frac{x(n)}{K(n)}\right)\right). \tag{1.2}$$

With the assumption that both $\{r(n)\}$ and $\{K(n)\}$ are bounded nonnegative sequences. Sufficient conditions for the persistence and existence of a stable positive periodic solution of system (1.2) were established.

Furthermore, under the assumptions of almost periodicity of the coefficients of system (1.2), Li and Chen [20] obtained the existence of a unique almost periodic solution which is globally attractive (see the following theorem).

Theorem 1.1 [20]. *Assume that*

$$\frac{K^M}{K^L}\exp\left(r^M - 1\right) < 2. \tag{H1}$$

Then almost periodic difference system (1.2) admits a unique almost periodic sequence solution which is globally attractive.

Chen and Zhou [21] extended system (1.2) to the following two species Lotka–Volterra competition system

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\left(r_1(n)\left(1 - \frac{x_1(n)}{K_1(n)} - \mu_2(n)x_2(n)\right)\right),\\ x_2(n+1) &= x_2(n)\exp\left(r_2(n)\left(1 - \mu_1(n)x_1(n) - \frac{x_2(n)}{K_2(n)}\right)\right) \end{aligned} \tag{1.3}$$

and obtained sufficient conditions which guarantee the persistence of system (1.3). Also, for the periodic case, they gave sufficient conditions which guarantee the existence of a globally stable periodic solution.

Li et al. [22] further incorporated the time delays in system (1.3), and considered the following system

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\left(r_1(n)\left(1 - \tfrac{x_1(n-\tau_{11})}{K_1(n)} - \mu_2(n)x_2(n-\tau_{12})\right)\right),\\ x_2(n+1) &= x_2(n)\exp\left(r_2(n)\left(1 - \mu_1(n)x_1(n-\tau_{21}) - \tfrac{x_2(n-\tau_{22})}{K_2(n)}\right)\right). \end{aligned} \tag{1.4}$$

Using almost periodic functional hull theory, the authors [22] showed that the almost periodic system is permanent, and has a unique strictly positive almost periodic solution, which is globally attractive. System (1.1) can be seen as the one species case of system (1.4), A similar analysis to Theorem 4.1 [22] yields the following theorem.

Theorem 1.2. *Assume that*

$$\eta = \min\left\{\frac{r^L}{K_M}, \frac{2}{M} - \frac{r^M}{K_L}\right\} - \tau\frac{r^M}{K_L}\left(\frac{r^M}{K_L}M + r^M\max\left\{1 - \frac{m}{K^M}, \left|1 - \frac{M}{K^L}\right|\right\}\right)\theta > 0, \tag{H2}$$

where $M,\ m$ are defined in Lemma 3.1 and $\theta = \exp\left(r^M\left(1 - \frac{m}{K^M}\right)\right)$. Then the almost periodic difference system (1.1) admits a unique almost periodic sequence solution which is globally attractive.

Obviously, condition (H_2) is complicated and is difficult to verify. Therefore, by constructing a suitable Lyapunov functional and using almost periodic functional hull theory, the main purpose of this paper is to derive a simple sufficient condition which guarantee the existence of a unique globally attractive almost periodic sequence solution of system (1.1). More precisely, we have the following theorem.

Theorem 1.3. *Assume that*

$$\frac{K^M}{K^L}\exp(r^M(1+\tau) - 1) < \frac{2}{1+2\tau}. \tag{H3}$$

Then the almost periodic difference system (1.1) admits a unique almost periodic sequence solution which is globally attractive.

Remark 1.1. By comparison, condition (H_3) is weaker than (H_2) for $\tau \geqslant 0$. Hence, Theorem 1.3 improves Theorem 1.2. When $\tau = 0$, condition (H_3) reduces to (H_1). Therefore, Theorem 1.3 generalizes the main results of [19,20].

For more works on almost periodic solution of almost discrete systems, one could refer to [23–31] and the references cited therein.

The organization of this paper is as follows. In Section 2, we give some definitions and present some useful lemmas. Sufficient conditions for the global attractivity of system (1.1) are obtained in Section 3. Then, in Section 4, we establish sufficient conditions for the existence of a unique almost periodic solution, which is globally attractive. The main result is illustrated by an example with numerical simulation in the last section.

2. Preliminaries

First, we give the definitions of the involved terminologies.