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Stability of a two-class two-server retrial queueing system

Bara Kim [a], Jeongsim Kim [b,*]

[a] Department of Mathematics, Korea University, 145 Anam-ro, Seongbuk-gu, Seoul, 136-701, Republic of Korea

[b] Department of Mathematics Education, Chungbuk National University, 1 Chungdae-ro, Seowon-gu, Cheongju, Chungbuk, 362-763, Republic of Korea

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ABSTRACT

We consider a two-class two-server retrial queueing system. Customers of each class arrive according to a Poisson process and the service times of each class of customers are assumed to be exponentially distributed with service rates depending on both the customer's class and the servers. We provide stability and instability conditions for this retrial queueing system.

1. Introduction

Retrial queues are queueing systems in which arriving customers who find all servers occupied may retry for service again after a random amount of time. Retrial queues have been widely used to model many problems in telephone systems, call centers, telecommunication networks, computer networks and computer systems, and in daily life. Detailed overviews for retrial queues can be found in the bibliographies [1–3], the surveys [4–6], and the books [7,8].

We consider a two-class two-server retrial queueing system. Customers of each class arrive according to a Poisson process and the service times of each class of customers are assumed to be exponentially distributed with service rates depending on both the customer's class and the servers. The following practical situation is an example that corresponds to this retrial queueing system. Consider a bilingual call center with two agents (servers) with different levels of language proficiency. There are two types of customers (callers) where one class is English speaking and the other is Spanish speaking. The two servers can speak English and Spanish. Server 1 speaks Spanish fluently but English not so good, whereas server 2 speaks English fluently but Spanish not so good.

The objective of this paper is to give sufficient conditions for ergodicity and transience of the Markov process describing the queueing system, which will be given in Section 2. Sufficient conditions for ergodicity and transience of the Markov process can be obtained by using criteria based on Lyapunov functions or test functions. A queueing system is called stable when the Markov process describing the queueing system is positive recurrent. For the stability analysis of queueing models, ergodicity and nonergodicity criteria with test functions have been used, see, for example, [9–11].

Deul [12] derived necessary and sufficient conditions for stability of M/M/m retrial queues. For the M/M/m retrial queues, Falin [13] presented a simpler method to obtain a sufficient condition for its stability through the use of Foster's criterion utilizing a test function of a very simple type. Another class of Lyapunov functions for the stability was introduced by

* Corresponding author. Tel.: +82 43 261 2716; fax: +82 43 275 2715.
E-mail addresses: bara@korea.ac.kr (B. Kim), jeongsimkim@chungbuk.ac.kr (J. Kim).

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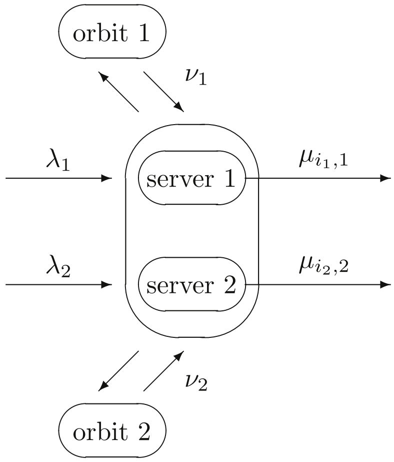

Fig. 1. Two-class two-server retrial queue ($i_1, i_2 = 1, 2$).

Hanschke [14,15]. Later, Falin [16] showed how criteria based on mean drift can be used to establish nonergodicity. For single-class multi-server retrial queue with phase-type retrial times, He, Li and Zhao [17] found a necessary and sufficient condition for stability by using test functions. Recently, Kim [18] found a necessary and sufficient condition for stability of multi-class multi-server retrial queue where the service times of each class of customers depend only on the server. Most researchers focus on the retrial queues in which the service rates are only server-dependent. But, the stability analysis of multi-class multi-server retrial queue with service rates of each class depending on both the customer's class and the servers, is much more difficult.

This paper is organized as follows. In Section 2, we introduce a Markov process describing the queueing system, and state the main theorem providing sufficient conditions for ergodicity and transience of the Markov process. Section 3 serves as preliminaries for the proof of the main theorem. In Sections 4 and 5, we give the proofs of sufficient conditions for ergodicity and transience by using drift analysis techniques.

2. The model and stability results

We consider the retrial queue consisting of two classes of customers (class 1 and class 2) and two servers (server 1 and server 2) as illustrated in Fig. 1. Customers of class i arrive from outside the system according to a Poisson process with rate $\lambda_i,\ i = 1, 2$. If there is a free server when class-i customer arrives from outside the system, this customer begins to be served by one of the servers immediately and leaves the system after the service is completed. On the other hand, any class-i customer who finds all the servers busy upon arrival joins orbit $i,\ i = 1, 2$, and then attempts service after a random amount of time. If there is a free server when a customer from orbit i attempts service, this customer receives service from one of the servers immediately and leaves the system after the service completion. Otherwise the customer comes back to orbit i immediately and repeats the retrial process. The service times of class-i customers by server $j,\ j = 1, 2$, are assumed to be exponentially distributed with mean μ_{ij}^{-1}, i.e., μ_{ij} is the service rate of class-i customers by server j. The inter-retrial times of each class-i customer in orbit i are exponentially distributed with mean ν_i^{-1}. The arrival process, the service times, and the retrial times are assumed to be mutually independent.

If both servers are idle when a class-i customer arrives from outside the system, then the customer enters server j with probability p_{ij}. If both servers are idle when a customer from orbit i attempts service, then the customer enters server j with probability $\widetilde{p}_{ij}$.

Let $N_1(t)$ be the number of customers in orbit 1 and $N_2(t)$ the number of customers in orbit 2. Let $K_j(t),\ j = 1, 2$, be the class of the customer who is being served by server j at time t if server j is busy at time t and 0 if server j is idle at time t. If we let $X(t) = (N_1(t), N_2(t), K_1(t), K_2(t)),\ t \geq 0$, then $\{X(t) : t \geq 0\}$ is a continuous time Markov process (CTMP) with state space $S \equiv S_{00} \cup S_{01} \cup S_{10} \cup S_{11}$, where $S_{00} = \{(n_1, n_2, 0, 0) : n_1, n_2 = 0, 1, 2, \ldots\}$, $S_{01} = \{(n_1, n_2, 0, k_2) : n_1, n_2 = 0, 1, 2, \ldots, k_2 = 1, 2\}$, $S_{10} = \{(n_1, n_2, k_1, 0) : n_1, n_2 = 0, 1, 2, \ldots, k_1 = 1, 2\}$, and $S_{11} = \{(n_1, n_2, k_1, k_2) : n_1, n_2 = 0, 1, 2, \ldots, k_1, k_2 = 1, 2\}$, and infinitesimal generator $Q = (q_{(n_1,n_2,k_1,k_2)(n_1',n_2',k_1',k_2')})$, $(n_1, n_2, k_1, k_2), (n_1', n_2', k_1', k_2') \in S$. The transition rates $q_{(n_1,n_2,k_1,k_2)(n_1',\, n_2',k_1',k_2')}$ are specified by the following:

(a) transitions from $(n_1, n_2, 0, 0) \in S_{00}$
- to $(n_1, n_2, 1, 0)$ with rate $\lambda_1 p_{11}$;
- to $(n_1, n_2, 0, 1)$ with rate $\lambda_1 p_{12}$;
- to $(n_1, n_2, 2, 0)$ with rate $\lambda_2 p_{21}$;
- to $(n_1, n_2, 0, 2)$ with rate $\lambda_2 p_{22}$;