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Computing finite time non-ruin probability and some joint distributions in discrete time risk model with exchangeable claim occurrences

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ABSTRACT

In this paper, we study a discrete time risk model based on exchangeable dependent claim occurrences. In particular, we obtain expressions for the finite time non-ruin probability, and the joint distribution of the time to ruin, the surplus immediately before ruin, and the deficit at ruin. An illustration of the results is given and some implications of the results are provided. Comparisons are made with the corresponding results for the classical compound binomial model of independent and identically distributed claim occurrences.

1. Introduction

Binomial models are used in finance and insurance on many topics like stock price analysis, pricing of financial derivatives, credit risk assessments and solvency oriented bankruptcy studies. For the last two, loss causing default events are the main concern in the modeling where the commonly envisaged important components are risk capital, income generating cash inflows, and default driven cash outflows. A general view of risk modeling on these topics can be found in the works of Kijima [1], Melnikov [2] and Franke et al. [3]. On the finance side, the loan and debt security markets are the vast grounds to observe default losses of debtors upon their failures in upfront scheduled payments. In the insurance market, claims of insureds are default events that may create loss burdens on insurers. In either case, each individual loss generating event can be modeled in probability with a Bernoulli distribution so that their aggregation in partial or complete sums follows a Binomial distribution when individual losses are independent. Time dynamic representations of such events are done with Binomial processes.

Concentrating on the insurance markets, the bankruptcy, as a solvency measure, for an insurer is modeled by a surplus or reserve process $\{U_n, n \in \mathbb{N}\}$ in order to express the evolution of surplus checked. When time is measured in discrete units, the process is a discrete one and the surplus at the end of time period n is defined by

$$U_n = u + n - \sum_{j=1}^{n} Y_j \tag{1}$$

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with $U_0 = u$, where u is the initial surplus, the periodic premium is one, and Y_j is the eventual claim amount in period j which is defined by

$$Y_j = \begin{cases} X_j, & I_j = 1 \\ 0, & I_j = 0, \end{cases}$$

where $I_j = 1$ if a claim occurs in period j and $I_j = 0$, otherwise, $j = 1, 2, \ldots$. The random variable I_j and the individual claim amount random variable X_j are independent in each time period. Eq. (1) can be equivalently defined as

$$U_n = u + n - \sum_{j=1}^{N_n} X_j, \tag{2}$$

where N_n is a random variable that stands for the number of claims up to time n. If the random indicators $I_1, I_2, \ldots$ are independent with $p = P\left\{I_j = 1\right\}$, then the above model is called the compound binomial model and

$$P\{N_n = k\} = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, \ldots, n.$$

There is a rich literature on the surplus processes in actuarial risk modeling. Consideration of the compound Binomial model for the surplus processes in analogy of the classical compound Poisson model is a relatively new approach. Gerber [4] investigates compound Binomial process modeling for the surplus processes. Then follow the studies on the subject by Michel [5], Shiu [6], Willmot [7], Dickson [8], Cheng et al. [9], Liu and Zhao [10], Lefèvre and Loisel [11], Wei and Hu [12]. Cheng et al. [9] studied the "discounted" probability $f(x, y; u)$ of ruin for an initial surplus u, such that the surplus just before ruin is x and the deficit at ruin is y under the compound binomial model. They have obtained both recursive and asymptotic formulas for $f(x, y; u)$. Liu and Zhao [10] derived the joint distribution of the time of ruin, the surplus immediately before ruin and the deficit at ruin in the compound binomial model. Recently, Li and Sendova [13] derived the probability distribution of the time to ruin under the compound binomial model following the idea in [14]. They have also obtained the distribution of the time the surplus process hits a specific level.

Define

$$T = \inf\{t > 0 : U_t \le 0\}$$

to be the random time to ruin. Let $\alpha(u; n) = P_u\{T > n\}$ denote the finite time non-ruin (survival) probability, $u > 0$. Then it can be computed recursively from

$$\alpha(u; n) = \sum_{t=1}^{n} pq^{t-1} \sum_{x=1}^{u+t-1} f(x)\alpha(u + t - x; n - t) + (1-p)^n, \tag{3}$$

for $n > 0$ with $\alpha(u; 0) = 1$ [4].

The assumption of independence between the claims occurrences may not be realistic for certain portfolios. Easing this assumption, Cossette et al. [15] studied the discrete time risk model (1) by assuming a Markovian type dependence between the claim occurrences. That is, $\{I_k, k \in \mathbb{N}\}$ is a Markovian process with a given transition probability matrix. Under this assumption, the model (1) is referred as the compound Markov binomial model. Cossette et al. [15] provided recursive formulas for computing the ruin probabilities over finite and infinite time horizons. Further computational extensions on this model is given by another work of Cossette et al. [16]. Yuen and Guo [17] extracted two discrete-time renewal risk processes from the compound Markov binomial model. They have investigated the Gerber–Shiu expected discounted penalty functions based on these renewal risk processes. Shizu et al. [18] obtained renewal equations for the conditional and unconditional Gerber–Shiu discounted penalty function under the Markov binomial model.

In this paper, we study the model (1) when the claim occurrences are exchangeable and dependent. The sequence of claim occurrence indicators $\{I_k, k \in \mathbb{N}\}$ is exchangeable if for each $n > 0$, the joint distribution of $I_1, I_2, \ldots, I_n$ is invariant under any permutation of its indices, i.e.

$$P\left\{I_{\pi_1} = x_1, I_{\pi_2} = x_2, \ldots, I_{\pi_n} = x_n\right\} = P\left\{I_1 = x_1, I_2 = x_2, \ldots, I_n = x_n\right\}$$

for any permutation $(\pi_1, \pi_2, \ldots, \pi_n)$ of the indices in $\{1, 2, \ldots, n\}$. Exchangeability in probability models is an important concept when joint distributions are sought for permutations of the elements of a sample itself or equal sized subsets of it without any judgement of independence or without requiring existence of any limit of relative frequencies. We refer to Schervish [19, pp. 5–52] for details on exchangeability and for some of its applications on parametric models. In this context, according to the fundamental Theorem of de Finetti, restated in the same reference, there is a random variable Θ supported on $(0, 1)$ with cumulative distribution function (c.d.f.) $G(\theta)$ such that $E(\Theta) = \theta$ and

$$P\{I_1 = x_1, \ldots, I_n = x_n\} = \int_0^1 \theta^k (1-\theta)^{n-k} dG(\theta),$$

where $k = \sum_{i=1}^{n} x_i$, and $G(\theta) = P\{\Theta \le \theta\}$ may be regarded as a mixing distribution. Such a dependence in $\{I_k, k \in \mathbb{N}\}$ occurs when the portfolio of an insurance company is diversified in the sense that the claim occurrence probabilities associated with