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Efficient valuation of SCR via a neural network approach

Seyed Amir Hejazi *, Kenneth R. Jackson

Department of Computer Science, University of Toronto, Toronto, ON, M5S 3G4, Canada

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ABSTRACT

As part of the new regulatory framework of Solvency II, introduced by the European Union, insurance companies are required to monitor their solvency by computing a key risk metric called the Solvency Capital Requirement (SCR). The official description of the SCR is not rigorous and has lead researchers to develop their own mathematical frameworks for calculation of the SCR. These frameworks are complex and are difficult to implement. Recently, Bauer et al. suggested a nested Monte Carlo (MC) simulation framework to calculate the SCR. But the proposed MC framework is computationally expensive even for a simple insurance product. In this paper, we propose incorporating a neural network approach into the nested simulation framework to significantly reduce the computational complexity in the calculation. We study the performance of our neural network approach in estimating the SCR for a large portfolio of an important class of insurance products called Variable Annuities (VAs). Our experiments show that the proposed neural network approach is both efficient and accurate.

1. Introduction

The Solvency II Directive is the new insurance regulatory framework within the European Union. Solvency II enhances consumer protection by requiring insurers to monitor the risks facing their organization. An integral part of Solvency II is the Solvency Capital Requirement (SCR) that reduces the risk of insurers' insolvency. SCR is the amount of reserves that an insurance company must hold to cover any losses within a one year period with a confidence level of 99.5%.

The calculation standards are described in the documents of the Committee of European Insurance and Occupational Pensions Supervisors (CEIOP) (e.g., [1]). The regulation allows insurance companies to use either the standard formula or to develop an internal model based on a market-consistent valuation of assets and liabilities. Because of the imprecise language of the aforementioned standards, many insurance companies are struggling to implement the underlying model and to develop efficient techniques to do the necessary calculations. In [2,3], rigorous mathematical definitions of SCR are provided. Moreover, [3] describes an implementation of a simplified, but approximately equivalent, notion of SCR using nested Monte Carlo (MC) simulations.

The results of the numerical experiments in [3] to find the SCR for a simple insurance product show that the proposed nested MC simulations are too expensive, even for their simplified notion of SCR. Hence, insurance companies cannot directly use the proposed MC approach to find the SCR for their large portfolios of insurance products. In this paper, we propose a neural network approach to be used within the nested MC simulation framework to ameliorate the computational complexity of MC simulations which allows us to efficiently compute the SCR for large portfolios of insurance products. We provide insights into the efficiency of the proposed extension of the MC simulation framework by studying its performance in computing the SCR for a large portfolio of Variable Annuities (VAs), a well-known and important class of insurance products.

* Corresponding author.
E-mail addresses: amir@cs.toronto.edu (S.A. Hejazi), krj@cs.toronto.edu (K.R. Jackson).

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A VA is a tax-deferred retirement vehicle that allows a policyholder to invest in financial markets by making payment(s) into a predefined set of sub-accounts set up by an insurance company. The investment of the policyholder should be payed back as a lump-sum payment or a series of contractually agreed upon payments over a period of time in the future. VA products provide embedded guarantees that protect the investment of a policyholder in a bear market and/or from mortality risk [4]. For a detailed description of VA products and the different types of guarantees offered in these products, see our earlier paper [5] and the references therein.

Because of the innovative structure of embedded guarantees in VA products, insurance companies have been successful in selling large volumes of these products [6]. As a result, VA products are a large portion of the investment market around the globe and big insurance companies have accumulated large portfolios of these products. The embedded guarantees of VA products expose insurers to a substantial amount of market risk, mortality risk, and behavioral risk. Hence, big insurance companies have developed risk management programs to hedge their exposures, especially after the market crash of 2008.

The rest of this paper is organized as follows. In Section 2, we describe the mathematical definition of SCR as well as its simplified, almost equivalent, version described in [3]. In Section 3, we describe a modification of the nested simulation approach of [3] that we use to approximate the SCR. Furthermore, we define a simple asset and liability structure that allows us to remove the assets from the required calculation of the SCR for the portfolio. In Section 4, we describe the neural network framework that we use to estimate the one-year probability distribution of liability for the input portfolio of VA products. In Section 5, we compare the efficiency and accuracy of our method to that of a simple nested MC simulation approach. In Section 6, we conclude the paper.

2. Solvency capital requirement

A rigorous treatment of SCR[1] requires the definition of Available Capital (AC) which is a metric that determines the solvency of a life insurer at each point in time. The AC is the difference between the Market Value of Assets (MVA) and Market Value of Liabilities (MVL):

$$\mathrm{AC}_t = \mathrm{MVA}_t - \mathrm{MVL}_t \tag{1}$$

where the subscript t denotes the time, in years, at which each variable is calculated.

Assuming the definition (1) of AC, the SCR, under Solvency II, is defined as the smallest amount of AC that a company must currently hold to insure a non-negative AC in one year with a probability of 99.5%. In other words, the SCR is the smallest amount x that satisfies the following inequality.

$$P(\mathrm{AC}_1 \geq 0 | \mathrm{AC}_0 = x) \geq 99.5\%. \tag{2}$$

In practice, it is hard to find the SCR using definition (2). Hence, Bauer et al. use a simpler, approximately equivalent notion of the SCR which is based on the one-year loss, Δ, evaluated at time zero:

$$\Delta = \mathrm{AC}_0 - \frac{\mathrm{AC}_1}{1+r} \tag{3}$$

where r is the one-year risk-free rate. The SCR is then redefined as the one-year Value-at-Risk (VaR):

$$\mathrm{SCR} = \mathrm{argmin}_x \{P(\Delta > x) \leq 0.5\%\}. \tag{4}$$

This is the definition of the SCR that we use in the rest of this paper.

3. Nested simulation approach

Given the formulation of Eq. (4), we can calculate the SCR by first computing the empirical probability distribution of Δ and then computing the 99.5%-quantile of the calculated probability distribution. We can implement this scheme by the nested simulation approach of [3]. In this section, we first outline the nested simulation approach of [3] and then describe our modification of it to make it more computationally efficient.

In the nested simulation approach of [3], summarized in Fig. 1, we first generate $N^{(p)}$ sample paths $\mathrm{P}^{(i)}$, $1 \leq i \leq N^{(p)}$, that determine the one-year evolution of financial markets. Note that we are only interested in the partial state of the financial markets. In particular, we are only interested in the state of the financial instruments that help us evaluate the asset values and the liability values of our portfolio. Hence, we can generate a sample state of the financial market by drawing one sample from the stochastic processes that describe the value of those financial instruments of interest.

In the nested simulation approach of [3], for each sample path $\mathrm{P}^{(i)}$, we use a MC simulation to determine the value $\mathrm{AC}_1^{(i)}$, the available capital one year hence. We also calculate AC_0 via another MC simulation and use that to determine the value of $\Delta^{(i)}$, $1 \leq i \leq N^{(p)}$, for each sample path $\mathrm{P}^{(i)}$, $1 \leq i \leq N^{(p)}$, via Eq. (3). The values $\Delta^{(i)}$, $1 \leq i \leq N^{(p)}$, can be used to determine the empirical distribution of Δ. In order to estimate the 99.5%-quantile for Δ as required by the definition of the SCR in Eq. (4), we can sort the calculated $\Delta^{(i)}$, $1 \leq i \leq N^{(p)}$, values in ascending order and choose the $\lfloor N \times 0.995 + 0.5 \rfloor$ element amongst the sorted values as the approximation for SCR.

[1] The material in this section is based largely on the discussion in [3].