Journal of Computational and Applied Mathematics 312 (2017) 65–73

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

# Sobolev-type fractional stochastic differential equations with non-Lipschitz coefficients

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ARTICLE INFO

*Article history:*
Received 29 September 2015
Received in revised form 21 December 2015

*MSC:*
26A33
65C30
47H10
93B05

*Keywords:*
Stochastic differential equations of Sobolev-type
Existence and uniqueness
Fractional derivatives
Non-Lipschitz coefficients

ABSTRACT

This paper investigates the existence and uniqueness of mild solutions for a class of nonlinear fractional Sobolev-type stochastic differential equations in Hilbert spaces. In this work, we used the fractional calculus, semigroup theory and stochastic analysis techniques for obtaining the required result. A new set of sufficient condition is established with the coefficients in the equations satisfying some non-Lipschitz conditions, which include classical Lipschitz conditions as special cases. More precisely, the results are obtained by means of standard Picard's iteration. Finally, an example is given to illustrate the obtained theory.



## 1. Introduction

Fractional differential equations attract many mathematicians and scientists because of their successful applications in various fields of science and engineering. Further, it can efficiently describe dynamical behavior of real life phenomena more accurately than integer order equations. Fractional differential equations have numerous applications in many areas such as in viscoelasticity, electrochemistry, nonlinear oscillation of earthquake, the fluid-dynamic traffic model, flow in porous media, aerodynamics and in different branches of physical and biological sciences [1–3]. Existence and uniqueness of mild solutions for deterministic fractional differential equations has been extensively studied in the literature (see [4,5] and the references therein). Wang et al. [6] studied the existence of mild solutions for the control system governed by fractional delay evolution inclusion in Banach spaces. The existence of mild solutions for fractional evolution equations with mixed monotone nonlocal conditions is studied in [7] by constructing a new monotone iterative technique.

In recent years, the theory and applications of stochastic differential equations in infinite-dimensional spaces have received much attention [8]. In particular, many mathematical models for dynamic processes in chemical, physical and biological sciences can be described by systems of stochastic differential equations [9]. Recently, the existence of mild solutions of stochastic differential evolution system in Hilbert spaces has been studied by many researchers (see [10,8] and references therein). The existence of mild solutions for a class of impulsive neutral stochastic integro-differential equations with infinite delays is studied in [11], where the Krasnoselskii–Schaefer type fixed point theorem combined with theories

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http://dx.doi.org/10.1016/j.cam.2015.12.020

of resolvent operators is used to obtain the required result. However, there has been little work in the area of fractional stochastic differential equations. Zhang et al. [12] considered a class of fractional stochastic partial differential equations with Poisson jumps, where a set of sufficient conditions for the existence and asymptotic stability in $p$th moment of mild solutions is obtained by employing a fixed point principle. Recently, Rajivganthi et al. [13] investigated the existence of mild solutions and optimal controls for a class of fractional neutral stochastic differential equations with Poisson jumps in Hilbert spaces, where a new set of sufficient conditions for the existence of mild solutions is obtained by using the successive approximation approach.

Sobolev-type equation appears in a variety of physical problems such as flow of fluid through fissured rocks, thermodynamics, propagation of long waves of small amplitude and so on [14]. In particular, Sobolev-type equation admits abstract representations in the form of implicit operator-differential equations with an arbitrary operator coefficient multiplying the highest derivative [15]. A detailed study of abstract Sobolev type differential equations has been discussed in the papers [16–18]. The existence result of mild solutions of fractional integrodifferential equations of Sobolev-type with nonlocal condition in a separable Banach space is studied by using the theory of propagation family as well as the theory of the measures of noncompactness and the condensing maps [18]. The existence and uniqueness of mild solution to Sobolev-type fractional nonlocal dynamical equations in Banach spaces is reported in [19], where a new set of conditions is obtained to achieve the required result by using fractional power of operators, a singular version of Gronwall's inequality and Leray–Schauder fixed point theorem.

On the other hand, many mathematical modeling of dynamical systems are based on the description of the properties of Sobolev-type equations. The Sobolev-type fractional models are more adequate than integer order models, so fractional order differential equations of Sobolev type have been investigated by many researchers ([20] and references therein). However, it should be mentioned that to the best of our knowledge, the existence and uniqueness of mild solutions of nonlinear fractional Sobolev-type stochastic differential equations in Hilbert spaces has not been investigated yet and this motivates our study. In order to fill this gap, in this paper, we study the existence and uniqueness of mild solutions for a class of nonlinear fractional Sobolev-type stochastic differential equations under non-Lipschitz conditions by employing Picard type approximate sequences. In particular, the obtained conditions are more general since it includes classical Lipschitz conditions as special cases. Finally, an example is provided to illustrate the obtained theory.

The rest of this paper is organized as follows. In Section 2, we will provide some basic definitions, lemmas of fractional calculus and stochastic analysis theory. Further, the problem formulation and the concept of mild solutions for the considered equations are provided. In Section 3, a set of sufficient conditions is obtained for the existence and uniqueness of mild solutions. In particular, the existence result is established by using stochastic analysis techniques, fractional calculation, semigroup theory and Picard type approximate sequences. In Section 4, an example is given to illustrate the obtained results.

## 2. Preliminaries

In this section, we provide some basic definitions, notations and lemmas, which will be used throughout the paper. In particular, we present main properties of semigroup theory [16], stochastic analysis theory [21,22] and well known facts in fractional calculus [3].

Let $H,\ E$ be two real separable Hilbert spaces and we denote by $\mathcal{L}(H, E)$ the space of all linear bounded operators from $H$ to $E$. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbf{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions, $W$ is a $Q$-Wiener process on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbf{P})$ with the linear bounded covariance operator $Q \in \mathcal{L}(E, E) = \mathcal{L}(E)$ such that $\mathrm{tr}Q < \infty$, where tr denotes the trace of the operator. Further, we assume that there exist a complete orthonormal system $\{e_n\}_{n\geq 1}$ in $E$, a bounded sequence of non-negative real numbers $\{\lambda_n\}$ such that $Qe_n = \lambda e_n$, $n = 1, 2, \ldots$ and a sequence $\{\beta_n\}_{n\geq 1}$ of independent Brownian motions such that

$$\langle w(t), e\rangle = \sum_{n=1}^{\infty} \sqrt{\lambda_n}\,\langle e_n, e\rangle\,\beta_n(t), \quad e \in E,\ t \in J \tag{1}$$

and $\mathcal{F}_t = \mathcal{F}_t^w$, where $\mathcal{F}_t^w$ is the $\sigma$-algebra generated by $\{w(s) : 0 \leq s \leq t\}$ and $\mathcal{F}_T = \mathcal{F}$. Let $\mathcal{L}_2^0 = \mathcal{L}_2(Q^{\frac{1}{2}}E; H)$ be the space of all Hilbert–Schmidt operators from $Q^{\frac{1}{2}}E$ to $H$ with the norm $\|\varphi\|_{\mathcal{L}_2^0} = \mathrm{tr}\,[\varphi Q\varphi^*] < \infty,\ \varphi \in \mathcal{L}(E, H)$. Let $L_2(\mathcal{F}_T, H)$ be the Hilbert space of all $\mathcal{F}_T$-measurable square integrable random variables with values in the Hilbert space $H$. Let $L_2^{\mathcal{F}}(J, H)$ is the Hilbert space of all square integrable and $\mathcal{F}_t$-adapted processes with values in $H$. Let $\mathbf{B}_T$ denote the Banach space of all $H$-valued $\mathcal{F}_t$-adapted processes $X(t, \omega) : J \times \Omega \to H$, which are continuous in $t$ for $a.e.$ fixed $\omega \in \Omega$ and satisfy

$$\|X\|_{\mathbf{B}_T} = \mathbf{E}\left(\sup_{t\in[0,T]} \|X(t, \omega)\|^p\right)^{1/p} < \infty, \quad p \geq 2. \tag{2}$$