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Analytical solutions for nonlinear long–short wave interaction systems with highly complex structure

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ABSTRACT

In this paper, we investigate and use the new modified $\exp(-\Omega(\xi))$-expansion method, (MEM). We apply the new MEM to nonlinear long–short-wave interaction systems (NLSWIS). Among our findings are sets of solutions including, but not limited to, new hyperbolic, complex, and dark soliton solutions. Not only is MEM shown to be highly adaptable for partial differential equations with strong nonlinearities, but also, it turns out to be highly efficient, despite its ease.

1. Introduction

Certain is it that nonlinear long–short-wave interaction systems, (NLSWIS), do mask inherently significant nonlinear processes, predictive of highly complex physical phenomena. NLSWIS processes do model nonlinear dynamical interaction between low-frequency long waves, and high-frequency short waves [1]. Highly motivating is uncovering basic physical interactions leading to further study and investigation of various nonlinear interactions underlying the general solution structure including: analytical, dark, and approximate solutions. The formation of special nonlinear waves such as solitons/soliton-like structures, shock waves, rogue waves, and vortex solitons is symbolized with the help of nonlinear wave interactions in the general forms of physical systems [2]. NLSWIS's are not only rich complex models for strong nonlinearities, but also represent the constitutive prototypes for highly interesting interaction phenomena, emanating from various applications, such as gravity and water waves, plasma and bio-physics, as well as nonlinear optics, to name a few, [3]. For instance, long gravity wave and capillary–gravity wave for finite-depth water interactions findings by Grimshaw in [4] are of major significance. In addition, his depiction of interactions between ultra-long equatorial waves, and short gravity waves in [5], remains notably significant in this regard. Alternative but very recent treatments of the regularized long wave equation can be found in Alam and Belgacem [6].

This paper is divided in the following manner: fundamental properties are provided in Section 2. In Section 3, we implement the MEM on the NLSWIS (1), and obtain new analytical solutions, related physical phenomena interpretation are discussed in Section 4. Relevant conclusions pertaining to the application of MEM to current NLSWIS models is done in Section 5.

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2. Fundamental properties of the MEM

To illustrate the efficiency of the new modified $\exp(-\Omega(\xi))$-expansion method, (MEM), we choose the multi-dimensional NLSWIS given in [7], related to physical conservation law with viscosity in [8], and introduced by Benney in [9],

$$\begin{aligned} &iu_t + u_{xx} - uv = 0,\\ &v_t + v_x + \left(|u|^2\right)_x = 0. \end{aligned} \tag{1}$$

This approach is based on the modified $\exp(-\Omega(\xi))$-expansion method [10–12]. In this section, we consider the following system of nonlinear partial differential equations [13–15];

$$\begin{aligned} &P_1(u, v, u_t, v_t, u_x, v_x, u_{xx}, v_{xx}, \ldots) = 0,\\ &P_2(u, v, u_t, v_t, u_x, v_x, u_{xx}, v_{xx}, \ldots) = 0, \end{aligned} \tag{2}$$

where, $u = u(x,t)$ and $v = v(x,t)$ are unknown functions, P_1 and P_2 are polynomials in $u(x,t)$ and $v(x,t)$, and then, its derivative where highest order derivatives and nonlinear terms are involved and the subscripts stand for the partial derivatives. The basic phases of MEM are expressed as follows:

Step 1: Combine the real variables x and t by a compound variable ξ

$$\begin{aligned} &u(x,t) = e^{i\eta}U(\xi), \qquad \eta = \alpha x + \beta t,\\ &v(x,t) = V(\xi), \qquad \xi = kx + wt. \end{aligned} \tag{3}$$

If we take the necessary derivations of Eqs. (3) for Eq. (1), we get the following expressions,

$$\begin{aligned} &u_x = ie^{i\eta}\alpha U + ke^{i\eta}U', \qquad u_t = ie^{i\eta}\beta U + we^{i\eta}U',\\ &u_{xx} = -\alpha^2 e^{i\eta}U + 2i\alpha k e^{i\eta}U' + k^2 e^{i\eta}U'',\\ &v_t = wV', \qquad v_x = kV', \qquad \left(|u|^2\right)_x = k\left(U^2\right)'\\ &\vdots \end{aligned} \tag{4}$$

where w, β are the frequencies of the travelling waves and α, k are the numbers of the waves. By using Eq. (4) in Eq. (2), we get the following nonlinear ordinary differential equation (NODE),

$$NODE(U, U', U'', U''', \ldots) = 0, \tag{5}$$

where, $NODE$ is a polynomial in U, and its ordinary derivatives with respect to ξ.

Step 2: We assume that the travelling wave solutions for Eq. (5) can be stated in the following form,

$$U(\xi) = \frac{\sum_{i=0}^{N} A_i\left[\exp(-\Omega(\xi))\right]^i}{\sum_{j=0}^{M} B_j\left[\exp(-\Omega(\xi))\right]^j} = \frac{A_0 + A_1\exp(-\Omega(\xi)) + \cdots + A_N\exp(-N\Omega(\xi))}{B_0 + B_1\exp(-\Omega(\xi)) + \cdots + B_M\exp(-M\Omega(\xi))}, \tag{6}$$

where, A_i $(0 \le i \le N)$ and B_j $(0 \le j \le M)$ are constants to be determined, such that $A_N \neq 0$, $B_M \neq 0$, and, $\Omega = \Omega(\xi)$ satisfies the following ordinary differential;

$$\Omega' = \mu\exp(\Omega) + \exp(-\Omega) + \lambda. \tag{7}$$

The following exact analytical solutions [13–15] can be written from Eq. (7):

Family-1: If $\mu \neq 0$, $\lambda^2 - 4\mu > 0$,

$$\Omega(\xi) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}}{2\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right), \tag{8}$$

where E is integral constant.

Family-2: When $\mu \neq 0$, $\lambda^2 - 4\mu < 0$,

$$\Omega(\xi) = \ln\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2\mu}\tan\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right). \tag{9}$$

Family-3: When $\mu = 0$, $\lambda \neq 0$, and $\lambda^2 - 4\mu > 0$,

$$\Omega(\xi) = -\ln\left(\frac{\lambda}{\exp(\lambda(\xi + E)) - 1}\right). \tag{10}$$