Journal of Computational and Applied Mathematics 312 (2017) 267–275

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

A logarithmic barrier approach for linear programming

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ARTICLE INFO

Article history:
Received 5 October 2015
Received in revised form 17 May 2016

MSC:
90C22
90C51

Keywords:
Linear programming
Interior point methods
Logarithmic barrier methods

ABSTRACT

This paper presents a logarithmic barrier method for solving a linear programming problem. We are interested in computation of the direction by the Newton's method and in computation of the displacement step using majorant functions instead line search methods in order to reduce the computation cost. This purpose is confirmed by numerical experiments, showing the efficiency of our approach, which are presented in the last section of this paper.

1. Introduction

Interior point methods are developed in the sixties by Dikin and Fiacco–McCormick [1], to solve nonlinear mathematical programs with large dimension. Some alternatives are conceived for the linear programming history to find the coherence between the theory and the practice, among powerful algorithms with polynomial complexity.

We distinguish three fundamental classes of interior point methods, namely: affine methods, potential reduction methods and central trajectory methods [2]. Interior point methods were the object of several research works, the ones done by Den Hertog [3], Nesterov and Nemirovski [4], Roos, Terlaky and Vial [5], Wright [2] and Ye [6].

Several algorithms have been proposed to solve the linear programming problem, by the projective interior point methods and their alternatives [7–10], central trajectory methods [11–13], logarithmic barrier methods [14]. Our work is based on the latter type of interior point methods. The main obstacle to establish an iteration is the determination and computation of the displacement step. Several alternatives are proposed to solve this problem. Unfortunately, the computation of displacement step, especially while using line search methods [15], is expensive and even more delicate in the case of semidefinite programming problems [14].

In semidefinite programming problems, effective and less expensive procedures are proposed by several researchers to avoid line search methods on one hand and to accelerate the convergence of algorithm on the other hand [16,14]. Our aim is to exploit this idea for linear programming.

Let us consider the following problem

$$(D)\begin{cases}\min b^t y\\ A^t y \geq c\\ y \in \mathbb{R}^m,\end{cases}$$

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http://dx.doi.org/10.1016/j.cam.2016.05.025
0377-0427/

where $A \in \mathbb{R}^{m\times n}$ such that $rangA = m < n$, $c \in \mathbb{R}^n$, $b \in \mathbb{R}^m$. The dual problem associated to (D) is the following linear program

$$(P)\begin{cases}\max c^t x\\ Ax = b\\ x \in \mathbb{R}^n,\ x \geq 0.\end{cases}$$

The problem (D) can be written in the following standard form

$$(D)\begin{cases}\min b^t y\\ A^t y - s = c\\ y \in \mathbb{R}^m,\ s \in \mathbb{R}^n \quad s \geq 0.\end{cases}$$

Let us note by

$S_D = \left\{y \in \mathbb{R}^m : A^t y - c \geq 0\right\}$, the feasible solutions set of (D).
$S_D^0 = \left\{y \in \mathbb{R}^m : A^t y - c > 0\right\}$, the strictly feasible solutions set of (D).
$S_P = \{x \in \mathbb{R}^n, Ax = b, x \geq 0\}$, the feasible solutions set of (P).
$S_P^0 = \{x \in \mathbb{R}^n, Ax = b, x > 0\}$, the strictly feasible solutions set of (P).

Let $u, v \in \mathbb{R}^n$, their scalar product is defined by

$$\langle u, v\rangle = u^t v = \sum_{i=1}^{n} u_i v_i.$$

We suppose that the sets S_D^0 and S_P^0 are not empty.

The solution of problem (D), is equivalent to the solution of a perturbed problems without constraints defined by

$$(D_r)\begin{cases}\min f_r\,(y)\\ y \in \mathbb{R}^m,\end{cases}$$

with $r > 0$ is a parameter barrier and f_r is a barrier function defined by

$$f_r\,(y) = \begin{cases} b^t y + nr\ln r - r\displaystyle\sum_{i=1}^{n} \ln\left\langle e_i, A^t y - c\right\rangle & \text{if } A^t y - c > 0,\\ +\infty & \text{Otherwise}\end{cases}$$

where $(e_1, e_2, \ldots, e_n)$ is the canonical base in $\mathbb{R}^n$. We are interested then in solving the problem (D_r). Indeed, we study in the first time in Section 2, the existence and uniqueness of the optimal solution of the problem (D_r). We show then that the problem (D_r) converges towards the problem (D), in the sense that if y_r is an optimal solution of (D_r), then $\lim_{r\longrightarrow 0} y_r = y^*$ is an optimal solution of (D).

In Section 3, we propose an interior point algorithm based on the Newton's approach which allows us to solve the nonlinear system resulting from the optimality conditions. The iteration of this algorithm is of descent type, defined by $y_{k+1} = y_k + t_k d_k$ where d_k is the descent direction and t_k is the displacement step. Also, we present different displacement steps by minimizing a majorant functions which approximate the unidimensional function $\varphi(t_k) = \min_{t>0} f(y + td)$.

Section 4, is dedicated to the presentation of comparative numerical tests to prove the performance of our approaches and to determine the most efficient algorithm.

2. Theoretical aspect of the problem (D_r)

2.1. Existence of solution of the problem (D_r)

Firstly, we give the following definition

Definition 2.1. Let f be a function defined from $\mathbb{R}^m$ to $\mathbb{R} \cup \{\infty\}$, f is called inf-compact if for all $r > 0$, the set $S_r\,(f) = \{x \in \mathbb{R}^m : f\,(x) \leq r\}$ is compact, which comes in particular to say that its cone of recession is reduced to zero.

To prove that (D_r) has an optimal solution, we show that f_r is inf-compact. For that, it is enough to prove that the cone of recession:

$$S_0\left((f_r)_\infty\right) = \left\{d \in \mathbb{R}^n, (f_r)_\infty\,(d) \leq 0\right\}$$

is reduced to the origin, i.e.,

$$(f_r)_\infty\,(d) \leq 0 \Longrightarrow d = 0,$$

where $(f_r)_\infty$ is defined by

$$(f_r)_\infty\,(d) = \lim_{t\to+\infty} \frac{f_r\,(y + td) - f_r\,(y)}{t} = b^t d.$$

This needs to prove the following lemma.