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Computational testing algorithmic procedure of assessment for lifetime performance index of products with Weibull distribution under progressive type I interval censoring

Shu-Fei Wu *, Meng-Ju Lin

Department of Statistics, Tamkang University, Tamsui, Taipei, 25137, Taiwan, ROC

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ABSTRACT

The assessing of the lifetime performance is a very important topic in manufacturing or service industries. Process capability indices had been widely used to evaluate the process performance to the continuous improvement of quality and productivity. The lifetimes of products are assumed to have Weibull distribution with a known shape parameter and the larger-the-better lifetime performance index is considered. The maximum likelihood estimator is used to estimate the lifetime performance index based on the progressive type I interval censored sample. The asymptotic distribution of this estimator is also investigated. We use this estimator to develop the new hypothesis testing algorithmic procedure with respect to a lower specification limit. Finally, two practical examples are given to illustrate the use of this testing algorithmic procedure to determine whether the process is capable.

1. Introduction

In the competitive market, the quality of products is the main concern for most consumers. For measuring the quality of products, process capability indices (PCIs) had been widely used such as C_p, C_{pk}, C_{pm} and C_{pmk}. See more examples and details in Montgomery [1]. Since the lifetime of products is a larger-the-better type quality characteristic, the PCI of unilateral tolerance, C_L is consider to assess the performance of lifetime with Weibull distribution. For example, Tong et al. [2] constructed the uniformly minimum variance unbiased estimator (UMVUE) of C_L and built a hypothesis testing procedure under the assumption of one-parameter exponential distribution for the complete sample.

However, in practice, the experimenter may not always be in a position to observe the lifetimes of all the items on test. This may be because of time limitation and/or other restrictions (such as money, material resources, negligence of typist or recorder, mechanical or experimental difficulties, etc.) on data collection. Therefore, some incomplete data could be collected, such as progressive censoring data (see Balakrishnan and Aggarwala [3], Aggarwala [4], Wu et al. [5], Wu et al. [6], Sanjel and Balakrishnan [7], Lee et al. [8], and Wu [9]). Ahmadi et al. [10] estimated the lifetime performance index with Weibull distribution based on progressive first-failure censoring scheme.

Wu and Lin [11] constructed the maximum likelihood estimator (MLE) for C_L and provided a hypothesis testing procedure for the one-parameter exponential distribution based on a progressive type I interval sample. The progressive type I interval censoring scheme is described as follows: Suppose that there are n products put on a life test at time 0. Let $(t_1, \ldots, t_m)$ be the predetermined inspection times, where t_m is scheduled to terminate the experiment. At time t_i, p_i is the pre-specified removal percentage of the remaining survival units at time t_i, $i = 1, \ldots, m$, where $p_m = 1$. During the first time interval

* Correspondence to: Department of Statistics, Tamkang University, Tamsui, Taipei, 25137, Taiwan, ROC. Fax: +886 2 26209732.
E-mail address: 100665@mail.tku.edu.tw (S.-F. Wu).

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$(0, t_1]$., the number of failure units X_1 is observed, and then $R_1 = [(n - X_1)\, p_1]$ units are randomly removed from the remaining $n - X_1$ survival units, where $[w]$ denotes the largest integer which is smaller than or equal to w. During the second time interval $(t_1, t_2]$, the number of failure units X_2 is observed and then $R_2 = [(n - X_1 - R_1 - X_2)\, p_2]$ units are randomly removed from the remaining $n - X_1 - R_1 - X_2$ survival units. Repeat the same process until the last time interval $(t_{m-1}, t_m]$, the number of failure units $X_m$is observed during and the remaining $R_m = n - \sum_{i=1}^{m} X_i - \sum_{i=1}^{m-1} R_i$ units are all removed. Then the life test is terminated at time t_m. The Weibull distribution with known shape parameter has been considered in literature and applied in practice (see for example [12–14]). Note that the widely used lifetime distributions, exponential and Rayleigh distributions, are special cases of this distribution. In this paper, we consider to investigate the estimation of C_L and build a hypothesis testing procedure to assess the lifetime performance of Weibull distribution, where the shape parameter is pre-determined by the maximum p-value method proposed by Lee [15] and hence supposed that the shape parameter is known.

The rest of this paper is organized as follows: In Section 2, some properties of the lifetime performance index for products with Weibull lifetimes are introduced and the relationship between the lifetime performance index and conforming rate is discussed. The MLE of the lifetime performance index and its related statistical properties based on the progressive type I interval censored sample from Weibull distribution are obtained in Section 3. In Section 4, we develop an algorithmic of hypothesis testing procedure for the lifetime performance index. Power analysis of the proposed testing procedure is also conducted in this section. Two numerical examples are given to illustrate our proposed testing procedure in Section 5. Finally, the conclusion is made in Section 6.

2. The lifetime performance index and the conforming rate

Suppose that the lifetime (U) of products follows a Weibull distribution with the probability density function (pdf) and the cumulative distribution function (cdf) as follows:

$$f(u) = \frac{\beta}{\lambda}\left(\frac{u}{\lambda}\right)^{\beta-1} e^{-(\frac{u}{\lambda})^{\beta}}, \quad u > 0, \beta > 0, \lambda > 0 \tag{1}$$

and

$$F(u) = 1 - \exp\left\{-\left(\frac{u}{\lambda}\right)^{\beta}\right\}, \quad u > 0, \beta > 0, \lambda > 0 \tag{2}$$

where λ is the scale parameter and β is shape parameter and the failure rate function is defined as

$$h(u) = \frac{f_U(u)}{1 - F_U(u)} = \frac{\beta}{\lambda}\left(\frac{u}{\lambda}\right)^{\beta-1}. \tag{3}$$

Montgomery [1] developed a process capability index C_L to measure the larger-the-better type quality characteristics as follows:

$$C_L = \frac{\mu - L}{\sigma}, \tag{4}$$

where μ denotes the process mean, σ represents the process standard deviation, and L is the known lower specification limit. This lifetime performance index can be used to assess the performance of lifetime of products. Using the transformation $Y = U^{\beta}$, then the new lifetime variable Y follows an exponential distribution with scale parameter $\tilde{k}$, where $\tilde{k} = k^{-1}$ and $k = \frac{1}{\lambda^{\beta}}$. The pdf and cdf of the new lifetime variable Y are given as follows:

$$f(y) = k\exp\{-ky\}, \quad y > 0, k > 0 \tag{5}$$

and

$$F(y) = 1 - \exp\{-ky\}, \quad y > 0, k > 0. \tag{6}$$

The mean and standard deviation of new lifetime variable Y are given by $\mu = E(Y) = \frac{1}{k}$ and $\sigma = \sqrt{Var(Y)} = \frac{1}{k}$. If L_U is the lower specification limit for lifetime variable U, then $L = L_U^{\beta}$ is the corresponding lower specification limit for new lifetime variable Y. The lifetime performance index C_L can be rewritten as

$$C_L = \frac{\mu - L}{\sigma} = \frac{1/k - L}{1/k} = 1 - kL. \tag{7}$$

We can see that the index $C_L > 0$ when $\lambda > L$ and $C_L < 0$ when $\lambda < L$. We can also see that the smaller the failure rate k the larger the lifetime performance index C_L. Therefore, the lifetime performance index C_L can accurately assess the lifetime performance of products.

An item of product is identified as a conforming one if its lifetime exceeds the lower specification limit (i.e. $Y \geq L$) and hence the conforming rate is defined as

$$P_r = P(Y \geq L) = \exp\{-kL\} = \exp\{C_L - 1\}, \quad -\infty < C_L < 1. \tag{8}$$