



A new characterization of the power distribution



G. Arslan*

Izmir University of Economics, Turkey

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ABSTRACT

A new characterization for the power function distribution is obtained which is based on products of order statistics. This result may be considered as a generalization of some recent results for contractions. The result is obtained by applying a new variant of the Choquet–Deny theorem. We note that in this new result the product consists of order statistics from independent samples. This characterization result may also be interpreted in terms of some special scheme of ranked set sampling.

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1. Introduction

In recent years one can find many different and interesting characterization results in the literature. For some particular examples, one may refer to [1–6], among others.

The power distribution has applications in finance and economics and is used to model reliability growth of complex systems or reliability of repairable systems (see, for example, [7,8]). In this paper, a new characterization of the power distribution, based on independent order statistics, is obtained. An interesting point of this new characterization result is that it is based on order statistics from independent sampled sets. In this respect, the obtained result may be considered as one of the first characterization results obtained by using order statistics from independent samples. We also note that the proof is given by using a new variant of the Choquet–Deny theorem (see, for example, [9]).

The paper is organized as follows. In Section 2, we introduce some basic notation and related results from the literature. In Section 3, ranked set samples are briefly introduced in order to express the characterization result with an appropriate notation. Then in the next section the main result is presented.

2. Basic notation and preliminaries

Consider three independent random variables X , Y , and U , where U has some known distribution. There are several recent characterization results, which may be considered as special cases of relation (1):

$$X \stackrel{d}{=} YU. \quad (1)$$

In its most basic form U can be assumed to have a uniform distribution concentrated on $(0, 1)$. In this case, relation (1) is an example of a contraction. These type of relations have some applications like in economic modeling and reliability, for example. Some of the first results of this type were obtained, among others, by [10–12].

We will write $X \sim \text{Pow}(\alpha)$ if $F_X(x) = x^\alpha$, $\alpha > 0$, $x \in (0, 1)$, and $Y \sim \text{Par}(\alpha)$, if $F_Y(y) = 1 - y^{-\alpha}$, $\alpha > 0$, $y \in (1, \infty)$. Let $X_{1:n}, X_{2:n}, \dots, X_{n:n}$ denote the order statistics for random variables X_1, X_2, \dots, X_n .

* Tel.: +90 2324889858.

E-mail addresses: guvenc.arslan@ieu.edu.tr, guvenc.arslan@gmail.com.

There are many interesting distributional relations and characterization results based on the relation given by (1). Wesolowski and Ahsanullah [3], for example, showed that $X_{i:n} \stackrel{d}{=} VX_{i-1:n}$ is a characteristic relation for the power distribution. Oncel et al. [13] investigated characterizations of the form $X_{i:n-1} \stackrel{d}{=} VX_{i:n}$, where V is a Pareto distribution. Navarro [14] obtained characterizations of $X_{i:n} \stackrel{d}{=} VX_{j:n}$, $1 \leq i < j \leq n$, where V is a power distribution. Martinez et al. [15] approached the same problem in a more general setting and used integral equations directly to solve the functional equations. In this way they also obtained some new characterization results.

We note that in all of the above characterizations one of the random variables is assumed to be known. In this paper, we will consider a new characterization involving products of order statistics, which may be considered to be similar to the relation (1). The main difference of this characterization from the previous results given in the literature is that there is no term with a known distribution. In this respect it may be considered as one of the first characterization results of the form

$$X \stackrel{d}{=} YZ \quad (2)$$

where all three random variables X , Y , and Z are independent random variables with unknown distribution functions.

3. Ranked set samples and independent order statistics

Before stating the main result, it will be convenient to introduce ranked set samples (RSS). Ranked set sampling is an alternative sampling design to simple random sampling when actual measurement is either difficult or expensive, but ranking a few units in a small set is relatively easy and inexpensive. This sampling design was first introduced by [16,17].

An RSS can be described as follows. Let $X_1, X_2, \dots, X_n, \dots$ be independent and identically distributed random variables with cdf F . Consider r independent sets of samples of sizes n_1, n_2, \dots, n_r , from this distribution where $r \leq n_r$. From these sets of independent samples we select r random variables as follows. From the first set of n_1 independent variables we select the smallest ($X_{1:n_1}^{(1)}$), while from the second set we select the second smallest ($X_{2:n_2}^{(2)}$). In this way we continue to select independent random variables until we have selected r representative random variables denoted here by $X_{[1,n_1]}, X_{[2,n_2]}, \dots, X_{[r,n_r]}$. The notation $X_{[i,n_j]}$, $1 \leq j \leq r$ is used to express the fact that each ordered random variable is selected from independent sets as described. In this way a set of independent order statistics is obtained. We note here also that the basic idea in RSS is to rank the observations in each set without actual measurement. This process has been summarized as follows:

$$\begin{array}{ccccccc} X_{1:n_1}^{(1)} & X_{2:n_1}^{(1)} & \dots & X_{n_1:n_1}^{(1)} & \rightarrow & X_{[1,n_1]} & \sim F_{1:n_1}(x) \\ X_{1:n_2}^{(2)} & X_{2:n_2}^{(2)} & \dots & X_{n_2:n_2}^{(2)} & \rightarrow & X_{[2,n_2]} & \sim F_{2:n_2}(x) \\ \dots & \dots & \dots & \dots & \rightarrow & \dots & \\ X_{1:n_r}^{(r)} & X_{2:n_r}^{(r)} & \dots & X_{n_r:n_r}^{(r)} & \rightarrow & X_{[r,n_r]} & \sim F_{r:n_r}(x). \end{array}$$

If the number of elements in each set is the same, i.e. $n_i = n$ for $1 \leq i \leq r$, and $r \leq n_r$, the obtained sample is called a balanced RSS. The joint pdf of $X_{[1,n_1]}, X_{[2,n_2]}, \dots, X_{[r,n_r]}$ is given by

$$f_{[1,2,\dots,r]}(x_{[1]}, x_{[2]}, \dots, x_{[r]}) = \prod_{i=1}^r f_{i:n_i}(x_{[i]}),$$

where $f_{i:n_i}(x)$ is the pdf of the i -th order statistic for a simple random sample of size n_i . The extra information provided by the structure of the ranking process and the independence of the obtained order statistics enables RSS to improve some of the classical approaches based on simple random sampling. For more information on RSS and its applications one may refer to [18,17,19], among others.

In the following we actually will use an unbalanced RSS. In particular, we will use the following three sets of independent random variables

$$\begin{array}{ccccccc} X_{1:n}^{(1)} & \dots & X_{k:n}^{(1)} & \dots & X_{n-1:n}^{(1)} & X_{n:n}^{(1)} & \rightarrow X_{[k,n]} \sim F_{k:n}(x) \\ X_{1:n-1}^{(2)} & \dots & X_{k:n-1}^{(2)} & \dots & X_{n-1:n-1}^{(2)} & & \rightarrow X_{[k,n-1]} \sim F_{k:n-1}(x) \\ X_{1:n}^{(3)} & \dots & X_{k:n}^{(3)} & \dots & X_{n-1:n}^{(3)} & X_{n:n}^{(3)} & \rightarrow X_{[n,n]} \sim F_{n:n}(x) \end{array}$$

to obtain the three independent order statistics $X_{[k,n]}$, $X_{[k,n-1]}$, and $X_{[n,n]}$.

4. Results

To prove the main result, the following lemma will be used. We note that this lemma can be considered as another special variant of the Choquet–Deny Theorem. For other variants and some applications of this theorem one may refer to [9,4,5], among others. This lemma can be proved by using the same idea as in the proof of Theorem 1 in [9].

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