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Topographical global initialization for finding all solutions of nonlinear systems with constraints

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ABSTRACT

We apply a recently revisited version of the topographical global initialization to solve nonlinear systems of equations with multiple roots subject to inequality constraints. This initialization technique is a simple and ingenious approach based on elementary concepts of graph theory. Here, the topographical initialization is used to generate good starting points to solve constrained global minimization problems, whose solutions are roots of associated nonlinear systems. To accomplish the task of local search, in the minimization step we use a well-established interior-point method. Our methodology was compared against other methods using benchmarks from the literature. Results indicated that the present approach is a powerful strategy for finding all roots of nonlinear systems.

1. Introduction

We are interested in the problem of finding all solutions of nonlinear systems of equations with constraints. We express this problem as

$$F(x) = 0, \quad x \in \Omega \subset \mathbb{R}^n, \tag{1}$$

where $F : \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear mapping, which is supposed, for technical reasons, to be enough differentiable. The set $\Omega \subset \mathbb{R}^n$ is given by

$$\Omega = \left\{x \in \mathbb{R}^n; g_i(x) \le 0, \forall i = 1, \dots, m\right\}, \tag{2}$$

where $g_i(x)$ are real and smooth functions, which are not necessarily convex.

There is a clear relationship between finding $x \in \Omega \subset \mathbb{R}^n$, such that $F(x) = 0$, and solve the following constrained minimization problem:

$$\operatorname{Min} f(x) = \left\|F(x)\right\|^2, \quad x \in \Omega \subset \mathbb{R}^n, \tag{3}$$

where $\|\cdot\|$ denotes the Euclidean norm. In fact, $x \in \Omega$ is a solution of (1) if and only if $x \in \Omega$ is a global minimizer of (3). Here, following Nocedal and Wright [18], a point $x \in \Omega$ is a global minimizer of f in Ω if $f(x) \le f(y)$, for all $y \in \Omega$.

Classical methods used to solve constrained optimization problems are developed from the mathematical analysis in $\mathbb{R}^n$, which is a theory with local characteristics. Thus, such iterative algorithms perform only local search around a given starting

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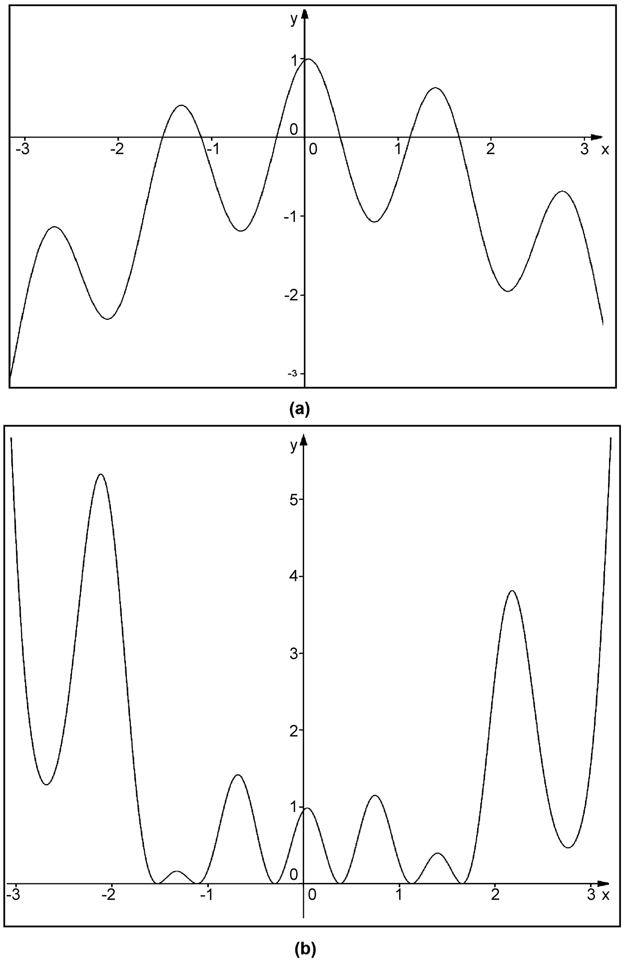

Fig. 1. (a) The plane curve described by a nonlinear equation $F(x) = 0$, and (b) the graph of the associated merit function $f(x) = |F(x)|^2$.

point. Hence, unless they are started close to the solutions of $F(x) = 0$, these local search methods do not give preference to global minimizers of $f(x)$. This represents a severe limitation to the direct use of such methods for solving (3), where the determination of global minimizers is a natural requirement. Moreover, as is well known, the merit function $f(x) = \|F(x)\|^2$ generally tends to be a highly multimodal function [3,18]. To illustrate this fact, Fig. 1a shows the plane curve described by the following nonlinear equation

$$F(x) = \sin(4.5x - 30) - \left(\frac{x}{2} - 0.1\right)^2 = 0, \tag{4}$$

which has six roots in $[-3, 3]$. On the other hand, Fig. 1b shows the graph of the merit function $f(x)$, where we can observe the existence of eight local minimizers in the same interval $[-3, 3]$. In view of this multimodal behavior, classical methods may not even be able to ensure convergence to a single solution of $F(x) = 0$.

These difficulties have motivated different researchers in the search for new approaches to problem (1), which seek to find all the solutions of $F(x) = 0$ in a feasible set Ω, usually a box in $\mathbb{R}^n$. In fact, Gritton et al. [7] used homotopy continuation algorithms. Hansen and Sengupta [8] employed interval analysis [15]. Yamamura et al. [33] also proposed an algorithm based on interval analysis. Maranas and Floudas [17] use a branch and bound method. Methodologies based on metaheuristics for global optimization were tested by different authors [9,12,13,19,22,23]. Nonsmooth exclusion test was used by Stuber et al. [25]. Tsoulos and Stavrakoudis [30] applied methods such as Multistart and Minfinder. An algorithm based on convex mixed-integer program was presented by Tsai and Lin [28]. Yamamura and Fujioka [32] developed the dual simplex method.

The topographical global initialization is a simple and ingenious approach based on elementary concepts of graph theory, which was proposed by Törn [26,27] to generate good starting points for local search methods, from points distributed uniformly in the search space [1].

Originally this initialization strategy uses three steps: (i) A uniform sampling of N points in the search space Ω. (ii) The construction of the topograph, which is a graph with directed arcs connecting the sampled points on a k-nearest neighbors