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ScienceDirect
Journal of the Nigerian Mathematical Society 35 (2016) 66–81
Journal of the Nigerian Mathematical Society
www.elsevier.com/locate/jnnms

Effects of diffusion-thermo and thermo-diffusion on two-phase boundary layer flow past a stretching sheet with fluid-particle suspension and chemical reaction: A numerical study

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Received 25 August 2015; accepted 29 October 2015
Available online 27 November 2015

Abstract

A numerical investigation on laminar boundary layer flow, heat and mass transfer of two-phase particulate suspension induced by a linearly stretching sheet is carried out. In the mathematical formulation both the fluid and particle phases are treated as two separate interacting continua. The effects of magnetic field, diffusion-thermo, thermal-diffusion, thermal radiation and first order chemical reaction are taken into the account. The relevant governing partial differential equations corresponding to the momentum, energy and concentration are transformed into a system of non-linear ordinary differential equations with the help of appropriate similarity transformations and then solved numerically using Runge–Kutta-Fehlberg fourth fifth order method along with shooting scheme. The effects of the relevant physical parameters on the flow, heat and mass transfer characteristics of both fluid and particle phases were numerically obtained and discussed in detail. It is found that, the momentum, thermal and solute boundary layer thickness decreases with increasing the particles loading.

Keywords: Heat and mass transfer; Stretching sheet; Soret effect; Dufour effect; Numerical solution; Fluid-particle suspension

1. Introduction

The study of heat and mass transfer due to a stretching surface is of great practical importance to engineers and scientists because of its occurrence in many branches of science and engineering. It plays an important role in several engineering applications and manufacturing processes in the industry such as, aerodynamic extrusion of plastic sheets, manufacturing and rolling of plastic films, cooling of metal sheets and etc. Sakiadis [1] presented the pioneering work

Peer review under responsibility of Nigerian Mathematical Society.
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http://dx.doi.org/10.1016/j.jnnms.2015.10.003

on the boundary layer flow past a continues moving surface with a constant velocity. He formulated the boundary layer equations for two dimensional and symmetric flows. Crane [2] discussed boundary layer flow caused by the stretching of elastic flat surfaces under various physical situations. Grubka and Bobba [3] have worked on the heat transfer occurring on a linearly stretching surface under variable temperature.

The mass transfer caused by the temperature gradient is called Soret effect (thermal diffusion), while the heat transfer caused by the concentration gradient is called Dufour effect (diffusion-thermo). The Soret effect is utilised for isotope separation and in a mixture between gases with very light molecular weight and of medium molecular weight. The importance of Soret and Dufour effects for fluids has been investigated by Dursunkaya and Worek [4], Anghel and Takhar [5] and Postelnicu [6]. Hayat et al. [7] developed analytical solutions for Soret and Dufour effects on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with viscoelastic fluid. Beg et al. [8] numerically investigated the free convection magnetohydrodynamic heat and mass transfer from a stretching surface to a saturated porous medium with Soret and Dufour effects. Aziz [9] addressed the thermal-diffusion and diffusion-thermo effects on combined heat and mass transfer by hydromagnetic three-dimensional free convection over a permeable stretching surface. Pal and Chatterjee [10] discussed the mixed convection heat and mass transfer past a stretching surface in a micropolar fluid saturated porous medium under the influence Soret and Dufour effects. Makinde and Olanrewaju [11] studied the mixed convection with Soret and Dufour effects past a porous plate moving through a binary mixture of fluid. Recently, the same authors [12] have investigated the effects of thermal diffusion and diffusion thermo on MHD boundary layer flow of heat and mass transfer past a moving vertical plate.

Chemical reactions can be categorised as either homogeneous or heterogeneous processes. These reactions mainly depend on whether they occur in an interface or as a single phase volume reaction. Homogeneous reaction is one that occurs uniformly through a given phase whereas a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. In a first order reaction, the rate of reaction is directly proportional to the concentration. In many chemical engineering processes, a chemical reaction between a foreign mass and the fluid does occur. These processes take place in various industrial applications, such as production of polymer, manufacturing of ceramics or glassware, food processing and so on. Therefore, the study of heat and mass transfer with chemical reaction effect is given primary importance in chemical and hydrometallurgical industries.
Pal and Mondal [13] theoretically investigated the effects of chemical reaction on MHD non-Darcian mixed convective heat and mass transfer over a linear stretching sheet. Further, they have analysed the chemical reaction effect on heat and mass transfer over a non-linear stretching sheet [14]. The problem of free convection heat and mass transfer over a porous stretching surface in the presence of chemical reaction has been studied by Kandasamy et al. [15]. The influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in a porous media with Soret and Dufour effects was numerically investigated by Postelnicu [16]. The investigations dealing with chemical reaction effect on fluid flow with different physical conditions can be found in the works of Das [17], Mansour et al. [18], Makinde et al. [19], Chamkha and EL-Kabeir [20] and Gireesha et al. [21].

The previously mentioned articles are concerned with single-phase flow analysis. The heat and mass transfer of fluid-particle mixture is a subject which attracted many investigators due to its application in many engineering and natural processes. These include environmental and atmospheric pollution, nuclear reactor safety, filtration, sedimentation of particles on gas turbine blades, particulate deposition on semi-conductor wafers in the electronic industry and others. Its relevance is also seen in gas cooling systems, fluidized beds, combustion, crude oil purification, electrostatic precipitation, polymer technology, transport processes and petroleum industry. In view of these applications, Saffman [22] was the first who formulated the governing equations for dusty fluid and has studied the stability of the laminar flow of a dusty gas. Later on, mathematical analysis based on the continuum approach for steady laminar boundary layer flow and heat transfer of fluid-particle suspension with different aspects were developed by Chamkha [23,24]. The free-convection flow and heat transfer of two-phase flow over an infinite porous vertical plate was theoretically analyzed by Chamkha and Ramadan [25]. They have developed analytical solutions and found that an increase of particle loading results in a retardation of momentum of both fluid and particle phases. Vajravelu and Nayfeh [26] have addressed the hydromagnetic flow of a dusty fluid over a stretching sheet. Makinde and Chinyoka [27] presented numerical results for MHD transient flow and heat transfer of a dusty fluid in a channel with variable physical properties and Navier slip condition. Attia et al. [28] have studied the ion slip effect on the unsteady Couette flow and heat transfer of a dusty fluid in the presence of uniform suction and injection. Recently, Gireesha et al. [29–31] have studied the fluid-particle suspension over a stretching surface with different physical conditions.