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A linear time algorithm to compute square of interval graphs and their colouring

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Abstract

The square of a graph $G = (V, E)$, denoted by G^2, is a graph on the same vertex set $V(G)$ such that two vertices x and y are adjacent in G^2 if and only if there is a path of length one or two between x and y in G. In this article, a new linear time algorithm is presented to compute G^2 from G when G is an interval graph. Also a linear time algorithm is designed to find all the maximal cliques of G^2 from G. Application of square of interval graphs in the field of $L(h, k)$-labelling problem is also discussed. Finally, it is shown that $L(1, 1)$-labelling number of an interval graph can be computed in linear time.

Keywords: Interval graph; Square of graph; Clique; $L(1, 1)$-labelling

1. Introduction

The kth power of a graph G denoted by G^k is a graph having the same vertex set as G and the distance between two vertices in G^k is one if and only if the distance between these two vertices in G is at most k. Obviously, $G^1 = G$. Due to their interesting properties and wide range of applications, power graph has been widely studied in the past. Power graph can be applied in different fields like routing in network, quantum random walk in physics, etc. The problem of colouring of power of graphs has also been considered in the past where the power of some specific classes of graphs like planar graph [1] and chordal graphs [2] has been studied. Vertex colouring of power of graphs has been used to solve different problems like interleaving [3], distributing data storage [4], sphere packing [5], etc. Square of graphs are also very useful in the study of radio communication networks. Vertex colouring of square of graphs are used to solve $L(1, 1)$-labelling problem of graphs. Again $L(1, 1)$-labelling problem has wide range of applications in the field of radio communication, mobile networking, frequency assignment [6], etc. $L(1, 1)$-labelling problem is a particular case of $L(h, k)$-labelling problem for $h = k = 1$.

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1.1. $L(h, k)$-labelling problem

The definition of $L(h, k)$-labelling is as follows.

Definition 1. $L(h, k)$-labelling of a graph $G = (V, E)$ is a function f from V to the set of non-negative integers $\{0, 1, \ldots, \lambda\}$ such that $|f(x) - f(y)| \geq h$ if $d(x, y) = 1$ and $|f(x) - f(y)| \geq k$ if $d(x, y) = 2$, where $d(x, y)$ is the length of the shortest path (i.e. the number of edges) between the vertices x and y.

The span of $L(h, k)$-labelling is the difference between largest and smallest used labels. The minimum span over all possible labelling functions is denoted by $\lambda_{h,k}(G)$.

For different values of h and k different problems have been addressed by the researchers. Roberts [7] investigated the problem for the case of $h = k = 1$, i.e. $L(1, 1)$-labelling problem. Griggs and Yeh [8] studied the problem for the case of $h = 2$ and $k = 1$. Bertossi and Bonuccelli [9] introduced a kind of integer control code assignment in packet radio networks to avoid hidden collisions. This problem is equivalent to the $L(0, 1)$-labelling problem. Also, channel assignment in optical cluster-based networks can be modelled either as the $L(0, 1)$- or $L(1, 1)$-labelling problem. In general, channel assignment problems, with a channel defined as a frequency, a time slot, a control code, etc. can be modelled by an $L(h, k)$-labelling problem, for suitable values of h and k. In the literature, there are so many results related to $L(h, k)$-labelling problem [6] are available. In this paper, we focus our attention on $L(1, 1)$-labelling of interval graphs.

1.2. Interval graph

An undirected graph $G = (V, E)$ is said to be an interval graph if the vertex set V can be put into one-to-one correspondence with a set I of intervals on the real line such that two vertices are adjacent in G if and only if their corresponding intervals have non-empty intersection, i.e. there is a bijective mapping $f : V \to I$.

The set I is called an interval representation of G and G is referred to as the interval graph of I.

Interval graphs arise in the process of modelling many real life situations, specially involving time dependencies or other restrictions that are linear in nature. This graph and various subclass thereof arise in diverse areas such as archaeology, molecular biology, sociology, genetics, traffic planning, VLSI design, circuit routing, psychology, scheduling, transportation etc. Recently, interval graphs have found applications in protein sequencing, macro substitution, circuit routine, file organization, job scheduling, routing of two points nets and so on. In addition to these, interval graphs have been studied intensely from both the theoretical and algorithmic point of view. A brief discussion about interval representation of interval graphs and their properties are presented in Section 2.

1.3. Motivation of the work

Due to wide range of applications, $L(h, k)$-labelling problems has been widely studied over the lase two decades. In the algorithmic point of view the problem is NP-complete for general graphs. In case of interval graph, $L(0, 1)$-labelling problem is polynomially solvable [10] and the complexity is still open for $L(2, 1)$-labelling problem [11]. But, there is no such algorithm for $L(1, 1)$-labelling of interval graph. Motivated from these we studied $L(1, 1)$-labelling problem on interval graphs. To solve the problem we find the square of the given graph. Square of a graph play an important role in the field of graph theory. Note that the square of an interval graph is also an interval graph [12]. Thus designing a simple linear time algorithm to compute square of graphs is also the motivation of our work.

1.4. Our contribution

To the best of our knowledge there is no algorithm is available to compute the interval representation of G^2 from an interval representation of G. In this paper, a linear time algorithm is designed to compute interval representation of G^2. Also, a linear time algorithm is presented to compute all maximal cliques of G^2. A good relationship is established between $L(1, 1)$-labelling of a graph and colouring of square of graph. By using this relation we prove that $L(1, 1)$-labelling of interval graphs can be computed in linear time.