Available online at www.sciencedirect.com
ScienceDirect
AKCE International Journal of Graphs and Combinatorics 13 (2016) 76–84
AKCE International Journal of Graphs and Combinatorics
www.elsevier.com/locate/akcej

The forcing vertex detour monophonic number of a graph ☆

P. Titus*, P. Balakrishnan

Department of Mathematics, University College of Engineering Nagercoil, Anna University : Tirunelveli Region, Nagercoil - 629 004, India

Received 30 May 2014; accepted 18 February 2016
Available online 31 March 2016

Abstract

For any two vertices x and y in a connected graph G, an x–y path is a monophonic path if it contains no chord, and a longest x–y monophonic path is called an x–y detour monophonic path. For any vertex x in G, a set $S_x \subseteq V(G)$ is an x-detour monophonic set of G if each vertex $v \in V(G)$ lies on an x–y detour monophonic path for some element y in S_x. The minimum cardinality of an x-detour monophonic set of G is the x-detour monophonic number of G, denoted by $dm_x(G)$. A subset T_x of a minimum x-detour monophonic set S_x of G is an x-forcing subset for S_x if S_x is the unique minimum x-detour monophonic set containing T_x. An x-forcing subset for S_x of minimum cardinality is a minimum x-forcing subset of S_x. The forcing x-detour monophonic number of S_x, denoted by $f_{dm_x}(S_x)$, is the cardinality of a minimum x-forcing subset for S_x. The forcing x-detour number of G is $f_{dm_x}(G) = \min\{f_{dm_x}(S_x)\}$, where the minimum is taken over all minimum x-detour monophonic sets S_x in G. We determine bounds for it and find the same for some special classes of graphs. Also we show that for every pair s, t of integers with $2 \le s \le t$, there exists a connected graph G such that $f_{dm_x}(G) = s$ and $dm_x(G) = t$ for some vertex x in G.

Keywords: Detour monophonic path; Vertex detour monophonic number; Forcing vertex detour monophonic number

1. Introduction

By a *graph* $G = (V, E)$ we mean a non-trivial finite undirected connected graph without loops or multiple edges. The *order* and *size* of G are denoted by n and m respectively. For basic graph theoretic terminology we refer to Harary [1]. For vertices x and y in a connected graph G, the *distance* $d(x, y)$ is the length of a shortest $x - y$ path in G. An $x - y$ path of length $d(x, y)$ is called an $x - y$ *geodesic*. The *neighborhood* of a vertex v is the set $N(v)$ consisting of all vertices u which are adjacent with v. The *closed neighborhood* of a vertex v is the set $N[v] = N(v) \cup \{v\}$. A vertex v is an *extreme vertex* of G if the subgraph induced by its neighbors is complete.

The *closed interval* $I[x, y]$ consists of all vertices lying on some $x - y$ geodesic of G, while for $S \subseteq V$, $I[S] = \bigcup_{x,y \in S} I[x, y]$. A set S of vertices is a *geodetic set* if $I[S] = V$, and the minimum cardinality of a geodetic

Peer review under responsibility of Kalasalingam University.
☆ Research supported by DST Project No. SR/S4/MS: 570/09.
* Corresponding author.
E-mail addresses: titusvino@yahoo.com (P. Titus), gangaibala1@yahoo.com (P. Balakrishnan).

http://dx.doi.org/10.1016/j.akcej.2016.03.002

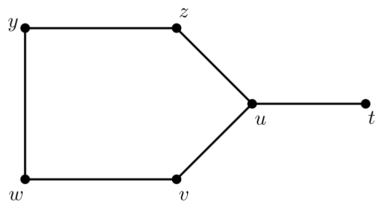

Fig. 2.1. The graph G in Example 2.2.

set is the *geodetic number* $g(G)$. A geodetic set of cardinality $g(G)$ is called a *g-set* of G. The geodetic number of a graph was introduced in [2] and further studied in [3–5].

The concept of vertex geodomination in graphs was introduced in [6] and further studied in [7]. Let x be a vertex of a connected graph G. A set S of vertices of G is an *x-geodominating set* of G if each vertex v of G lies on an $x-y$ geodesic in G for some element y in S. The minimum cardinality of an x-geodominating set of G is defined as the *x-geodomination number* of G and is denoted by $g_x(G)$.

A *chord* of a path P is an edge joining any two non-adjacent vertices of P. A path P is called a *monophonic path* if it is a chordless path. A longest $x-y$ monophonic path P is called an *x − y detour monophonic path*. The *monophonic distance* $d_m(u,v)$ from u to v is defined as the length of a longest $u-v$ monophonic path (or $u-v$ detour monophonic path) in G. The *monophonic eccentricity* $e_m(v)$ of a vertex v in G is $e_m(v) = max\{d_m(v,u) : u \in V(G)\}$. The *monophonic radius*, $rad_m G$ of G is $rad_m G = min\{e_m(v) : v \in V(G)\}$ and the *monophonic diameter*, $diam_m G$ of G is $diam_m G = max\{e_m(v) : v \in V(G)\}$. The monophonic distance was introduced in [8] and further studied in [9].

The concept of vertex detour monophonic number was introduced in [10]. Let x be a vertex of a connected graph G. A set S of vertices of G is an *x-detour monophonic set* of G if each vertex v of G lies on an $x-y$ detour monophonic path in G for some element y in S. The minimum cardinality of an x-detour monophonic set of G is defined as the *x-detour monophonic number* of G and is denoted by $dm_x(G)$. An x-detour monophonic set of cardinality $dm_x(G)$ is called a dm_x-set of G.

2. Forcing vertex detour monophonic number

Let x be any vertex of a connected graph G. Although G contains a minimum x-detour monophonic set there are connected graphs which may contain more than one minimum x-detour monophonic set. For example the graph G given in Fig. 2.1 contains more than one minimum x-detour monophonic set. For each minimum x-detour monophonic set S_x in a connected graph G there is always some subset T_x of S_x that uniquely determines S_x as the minimum x-detour monophonic set containing T_x. Such sets are called "vertex forcing subsets" and we discuss these sets in this section. Also, forcing concepts have been studied for such diverse parameters in graphs as the graph reconstruction number [11], the domination number [12], and the geodetic number [13].

Definition 2.1. Let x be a vertex of a connected graph G and let S_x be a minimum x-detour monophonic set of G. A subset T_x of S_x is called an *x-forcing subset* for S_x if S_x is the unique minimum x-detour monophonic set containing T_x. An x-forcing subset for S_x of minimum cardinality is a *minimum x-forcing subset* of S_x. The *forcing x-detour monophonic number* of S_x, denoted by $f_{dm_x}(S_x)$, is the cardinality of a minimum x-forcing subset of S_x. The *forcing x-detour monophonic number* of G is $f_{dm_x}(G) = min\{f_{dm_x}(S_x)\}$, where the minimum is taken over all minimum x-detour monophonic sets S_x in G.

Example 2.2. For the graph G given in Fig. 2.1, the minimum vertex detour monophonic sets, the vertex detour monophonic numbers, the minimum forcing vertex detour monophonic sets and the forcing vertex detour monophonic numbers are given in Table 2.1.

The next theorem immediately follows from the definition of x-detour monophonic number and forcing x-detour monophonic number of a graph G.

Theorem 2.3. *For any vertex x in a connected graph G, $0 \leq f_{dm_x}(G) \leq dm_x(G)$.*