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Lower Bounding Procedures for the Single Allocation Hub Location Problem

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Abstract

This paper proposes a new lower bounding procedure for the Uncapacitated Single Allocation p-Hub Median Problem based on Lagrangean relaxation. For solving the resulting Lagrangean subproblem, the given problem structure is exploited: it can be decomposed into smaller subproblems that can be solved efficiently by combinatorial algorithms. Our computational experiments for some benchmark instances demonstrate the strength of the new approach.

Keywords: Hub Location, Lagrangian relaxation, Lower bounds.

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1 Introduction

Consider a complete graph $G = (V, E)$, where $V = \{1, 2, \ldots, n\}$ corresponds to origins, destinations and possible hub locations, and E is the edge set. Let b_{ij} be the nonnegative transport cost per unit of flow from node i to node j, and W_{ij} be the amount of flow from node i to node j. The cost per unit of flow for each path P_{ij}^{kl} from an origin node i to a destination node j which passes hubs k and l respectively, is $\beta_1 b_{ik} + \alpha b_{kl} + \beta_2 b_{lj}$, where β_1, α, and β_2 are the nonnegative collection, transfer and distribution costs respectively. The Uncapacitated Single Allocation p-Hub Median Problem (USApHMP) consists of selecting p nodes as hubs and assigning the remaining nodes to these p hubs such that each non-hub node is assigned to exactly one hub node with the minimum overall cost.

The quadratic binary programming formulation for the (USApHMP) is:

$$
\begin{aligned}
\min \quad & \sum_i \sum_j \sum_k b_{ik}(\beta_1 W_{ij} + \beta_2 W_{ji}) x_{ik} + \sum_i \sum_j \sum_k \sum_l \alpha b_{kl} W_{ij} x_{ik} x_{jl} \\
\text{s.t.} \quad & \sum_k x_{ik} = 1 \qquad \forall i && (1) \\
& x_{ik} \leq x_{kk} \qquad \forall i, k && (2) \\
& \sum_k x_{kk} = p && (3) \\
& x_{ik} \in \{0, 1\} \qquad \forall i, k, && (4)
\end{aligned}
$$

where the binary variable x_{ik} indicates the allocation of node i to the hub located at node k. Constraints (1) indicate that non-hub node i is allocated to precisely one hub node. Constraints (2) enforce that node i is allocated to a hub node at k only if a hub is located at node k. Constraint (3) restricts the number of selected hubs to p.

To ease the argumentation, we define $C_{ik} = b_{ik}(\beta_1 \sum_j W_{ij} + \beta_2 \sum_j W_{ji})$ and $Q_{ikjl} = \alpha b_{kl} W_{ij}$. This allows us to write down the objective function in a more condensed form:

$$\sum_i \sum_k C_{ik} x_{ik} + \sum_i \sum_j \sum_k \sum_l Q_{ikjl} x_{ik} x_{jl}.$$

The USApHMP was first introduced in [9] as a quadratic binary program. Since then, many exact and heuristic algorithms have been proposed in the literature, e.g., by Campbell [3], Ernst and Krishnamoorthy [4], Skorin-Kapov et al. [10], and Ilić et al. [7]. Due to the quadratic nature of the problem,