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Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 52 (2016) 77–84 www.elsevier.com/locate/endm

A Game-theoretic Algorithm for Non-linear Single-Path Routing Problems

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Abstract

This paper is devoted to non-linear single path routing problems, which are known to be NP-hard even in the simplest cases. We propose a Best Response algorithm, based on Game Theory, providing single-path routings with modest relative errors with respect to optimal solutions, while being several orders of magnitude faster than existing techniques.

Keywords: best response, single-path routing, game theory, non-linear programming

1 Introduction

We consider single-path routing problems with an additive and non-linear objective function. These routing problems belong to the class of non-linear mathematical programs involving integer variables (actually, binary variables in our case). These mathematical programs are known to be extremely hard

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http://dx.doi.org/10.1016/j.endm.2016.03.011

to solve, both from a theoretical and from a practical point of view. Even in the simplest case with binary variables, quadratic function and equality constraints, they are known to be NP-hard ([3]).

Several approaches have been proposed to solve non-linear integer programming problems. Some transform the discrete problem into a continuous one (see for example [8]). Despite their qualities, those techniques do not scale very well with the size of the problems. A recent alternative is the so-called *Global Smoothing Algorithm* [7], which seems to scale better while providing fairly good approximations (see Section 3.1 for details).

Heuristics and meta-heuristics have also been used to solved non-linear integer programming problems. Among others, ant-inspired optimization techniques are known to be efficient for solving various routing problems ([10], [6]). In this paper, we use the heuristic method proposed in [6] for comparison purposes (see Section 3.2 for details).

We propose an approximation algorithm, inspired from Game Theory [5], for solving non-linear single-path routing problems. Indeed, we assume that individual flows are allowed to independently select their path to minimize their own cost function. We note that a similar algorithm was proposed in [1] for scheduling of strictly periodic tasks. As will be shown numerically, the main merit of this algorithm is that it is several orders of magnitude quicker than the method discussed above while providing good optimization results.

1.1 Problem statement

Consider a network represented by a directed graph $G = (V, E)$. To each edge $e \in E$ is associated a capacity c_e, and a non-decreasing, continuously differentiable and convex latency function $\ell_e : \mathbb{R}_+ \to \mathbb{R}_+$. For any set $\pi \subset E$, we define the constant δ^e_π as 1 if $e \in \pi$, and 0 otherwise.

We are given a set $\mathcal{K}$ of flows, each one associated with a traffic demand λ_k. We let s_k and t_k be the origin and destination of flow k. Each traffic flow has to be routed in the network over a single path. We let Π_k be the set of all paths available for routing traffic between s_k and t_k. A routing strategy for flow k is defined as a vector $\mathbf{x}_k \in \mathcal{S}_k = \left\{\mathbf{x}_k \in \{0,1\}^{|\Pi_k|} \;:\; \sum_{\pi\in\Pi_k} x_{k,\pi} = 1\right\}$, where $\mathcal{S}_k$ is the set of all feasible routing strategies for flow k, and the binary variable $x_{k,\pi}$ is 1 if flow k is routed over path π, and 0 otherwise. The vector $\mathbf{x} = (\mathbf{x}_k)_{k\in\mathcal{K}}$ will be called a routing strategy. It belongs to the product strategy space $\mathcal{S} = \bigotimes_{k\in\mathcal{K}} \mathcal{S}_k$. For each edge $e \in E$, we define the function $y_e : \mathcal{S} \to \mathbb{R}_+$ as the amount of traffic flowing through link e under strategy $\mathbf{x}$. We define the cost of a routing strategy $\mathbf{x} \in \mathcal{S}$ as $\phi(\mathbf{x}) = \sum_{e\in E} \ell_e\left(y_e(\mathbf{x})\right)$. We