Topology and its Applications 202 (2016) 1–6

Contents lists available at ScienceDirect

Topology and its Applications

www.elsevier.com/locate/topol

Absolute retracts and equiconnected monads

T. Radul [a,b,*,1]

[a] *Institute of Mathematics, Casimirus the Great University, Bydgoszcz, Poland*
[b] *Department of Mechanics and Mathematics, Lviv National University, Universytetska st., 1, 79000 Lviv, Ukraine*

ARTICLE INFO

Article history:
Received 18 February 2013
Received in revised form 1 May 2015
Available online 7 January 2016

MSC:
18B30
18C15
28A33
54C55
54B30

Keywords:
Monad
Equiconnectednes
Absolute retract
Probability measure
Idempotent measure

ABSTRACT

We introduce a general notion of equiconnected functor and show that each such functor has similar topological properties as probability measure functor and idempotent measure functor, which have some natural equiconnectedness structures.

0. Introduction

The general theory of functors acting in the category $\mathcal{C}omp$ of compact Haussdorff spaces (compacta) and continuous mappings was founded by E.V. Shchepin [1]. He distinguished some elementary properties of such functors and defined the notion of normal functor that has become very fruitful. The class of normal functors and similar classes includes many classical constructions: hyperspace exp, space of probability measures P, superextension λ, space of hyperspaces of inclusion G and many other functors [2,3].

The Wojdyslawski theorem (stating that the hyperspaces of Peano continua are absolute retracts [4]) can be considered as the start of investigation of functors in topological categories from the point of view of

* Correspondence to: Department of Mechanics and Mathematics, Lviv National University, Universytetska st., 1, 79000 Lviv, Ukraine.
E-mail address: tarasradul@yahoo.co.uk.
[1] This paper was presented at the International Conference on Topology and its Applications, Skopje 2012 (ICTA 2012 Conference).

http://dx.doi.org/10.1016/j.topol.2015.12.061

geometric topology. At last decades, there many results were obtained in this direction. The general problem was formulated in [2] by V.V. Fedorchuk: What properties of topological spaces are preserved or improved by functors?

Many functors have some algebraical structures. Such structures could be described by the notion of monad (or triple) structure in the sense of S. Eilenberg and J. Moore and their corresponding category of Eilenberg–Moore's algebras [5].

Many classical constructions lead to monads: hyperspaces exp, spaces of probability measures P, superextensions λ etc. There is a lot of investigations of topological properties of each such particular monad (see for example [6] or [3]). But there are also results which describe topological properties of arbitrary monads with some basic properties. For example it was shown in [7] and [8] that a monad $\mathbb{T} = (T, \eta, \mu)$ in the category $\mathcal{C}omp$ with certain additional assumption is binary if and only if TX is AE(0) compactum for each openly generated compactum X. Hence, we obtain a description of topological properties for a class of binary monads which includes such known monads as superextension monad, inclusion hyperspace monad, order preserving functional monad etc.

Let us consider probability measure functor P [9] and idempotent measure functor I [10] (both lead to corresponding monads). It is known that for each metric compacta the spaces PX and IX are absolute retracts. The space PX is absolute retract if and only if it is AE(0) which implies that PX is absolute retract for each openly generated compactum X with weight $\leq \omega_1$ [11]. The same property has the functor I [14]. For both functors the classes of absolute F-valued retracts coincide with the class $AE(0)$ (see [2] and [12]).

Various versions of equiconnectedness were used to investigate topological properties related to extension of continuous maps (see for example [15–18]). The main aim of this paper is to show that all before listed topological properties of the functors P and I are determined by some equiconnectedness structures which they naturally posses. To show this we introduce a general notion of equiconnected functor and monad and show that each such functor has all above mentioned topological properties.

The paper is organized as follows: in Section 1 we remind some categorical notions, introduce and investigate the notion of equiconnected functor, in Section 2 we show that each equiconnected functor has Milyutin property and in Section 3 we investigate topological properties of equiconnected monads.

1. Equiconnected functors

By $\mathcal{C}omp$ we denote the category of compact Hausdorff spaces (compacta) and continuous maps. In what follows, all spaces and maps are assumed to be in $\mathcal{C}omp$. All functors we consider are endofunctors in $\mathcal{C}omp$. A functor F is called continuous if it preserves the limits of inverse systems. A functor is called monomorphic if it preserves topological embeddings. For monomorphic functor F and an embedding $i : A \to X$ we shall identify the space $F(A)$ and the subspace $F(i)(F(A)) \subset F(X)$. For a functor F which preserves monomorphisms the *intersection-preserving* property is defined as follows: $F(\cap\{X_\alpha \mid \alpha \in \mathcal{A}\}) = \cap\{F(X_\alpha) \mid \alpha \in \mathcal{A}\}$ for every family $\{X_\alpha \mid \alpha \in \mathcal{A}\}$ of closed subsets of X. A functor is called seminormal if it is continuous, monomorphic, preserves empty space, one-point spaces and intersection. In what follows, all functors assumed to be seminormal. For such a functor there exists a unique natural embedding $\eta X : X \to FX$. Let us remark that for a continuous functor F the map $F : C(X, Y) \to C(FX, FY)$ is continuous for each compacta X and Y (we consider spaces $C(X, Y)$ and $C(FX, FY)$ with the compact-open topology) (see for example [3] for more details).

1.1. Definition. A functor F is called equiconnected if for each compactum X there exists a continuous map $\beta : X \times X \times [0, 1] \to FX$ such that for each x, $y \in X$ and $t \in [0, 1]$ we have $\beta(x, x, t) = \eta X(x)$, $\beta(x, y, 0) = \eta X(x)$, $\beta(x, y, 1) = \eta X(y)$ and $\beta(x, y, t) \in F(\{x, y\}) \subset FX$.