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A proof-theoretic universal property of determiners ☆

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ABSTRACT

The paper highlights a difference between Model-Theoretic Semantics (MTS) and Proof-Theoretic Semantics (PTS) regarding the meanings of NL-realisable determiners. While MTS uses conservativity as a major filter on GQs as serving NL-realisable determiners' meanings, conservativity fails in serving as such a filter. According to the PTS rendering of conservativity, all determiners are conservative. Instead of conservativity, PTS methodology focuses on other criteria, originating from the inferential role of determiners as captured by the introduction and elimination rules of a meaning-conferring ND-system. The criteria considered in the paper are harmony, stability and purity of rules. The paper presents two examples of conservative "determiners" that can be excluded by the suggested criteria.

1. Introduction

Determiners are the natural language analogue of quantifiers in logic. In model-theoretic semantics (MTS), their denotations are taken as binary relations over subsets of the domain of the model (see [9] for an extensive discussion). When combined with a noun-meaning, a subset of the domain, they give rise to *determiner phrases* (*dp*), which, according to the *generalised quantifiers* theory [1], a cornerstone of MTS, denote *generalised quantifiers* (GQs), subsets of the power set of the domain $\mathcal{D}$. It is generally *assumed* (and empirically attested), that the only GQs that can be denotation of *dp*s are the *conservative* ones, satisfying, for every $X, Y \subseteq \mathcal{D}$

$$(Cons1) \quad [\![D]\!](X)(Y) \text{ iff } [\![D]\!](X)(X \cap Y) \tag{1}$$

which in terms of denotations means that for every noun N and *vp* V:

$$(Cons2) \quad [\![D]\!]([\![N]\!])([\![V]\!]) \text{ iff } [\![D]\!]([\![N]\!])([\![N]\!] \cap [\![V]\!]) \tag{2}$$

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Expressed in terms of sentences, an example[1] is

$$\textsf{every girl smiles iff every girl is a girl who smiles} \tag{3}$$

Since there exist many non-conservative binary relations on subsets of a domain, conservativity serves as a *selection criterion* (a filter) for possible determiner denotations.

Why should it be a task of formal semantics to impose a filter on objects that can serve as denotations of NL-expressions (here – determiners)? By doing this filtering, the semantics imposes a "good access" of the determiner's meaning to the meanings of its arguments. Why is conservativity considered generally to be a "good" selection criterion? Because it says about a GQ Q (in a model M) that only the part of the extension of B (in that model) which is common to the extension of A matters for the truth of $Q^M(A, B)$ in that model. That is, the part $B - A$ doesn't matter. The part of the models that needs to be considered for determining truth is considerably reduced.

On the other hand, in *proof-theoretic semantics* (PTS), an approach to semantics according to which meaning is determined by means of the rules of a meaning-conferring natural-deduction (ND) proof-system; in particular, meanings are determined and specified independently of models and truth-conditions. Thus meaning is captured in PTS by *inferential role*. See [14] as a general reference for PTS and [4,6,3] for PTS for an extensional fragment of NL.

In [3], I suggest that the proof-theoretic meaning of a determiner D is the following (with some details suppressed):

$$(PDET) \quad [\![D]\!] = \lambda z_1 \lambda z_2 \lambda \Gamma. \bigcup_{\mathbf{j}_1,\ldots,\mathbf{j}_m \in \mathcal{P}} I_D(z_1)(z_2)(\mathbf{j}_1)\cdots(\mathbf{j}_m)(\Gamma) \tag{4}$$

where the notation means (with more detail below):

- z_1 ranges over noun proof-theoretic meanings and z_2 over *vp* proof-theoretic meanings.
- Γ is a collection of NL sentences (in the fragment under consideration), from which a conclusion sentence S that includes a *dp* headed by D (here, in subject position only, for simplicity) can be inferred.
- The $\mathbf{j}_k$s are *individual parameters*.
- I_D is the introduction-rule (I-rule) for D in the meaning-conferring ND-system. The *dp* headed by D is introduced into the subject position of S similarly to introducing a connective or quantifier into a logical formula.

Based on this characterisation of determiners' meanings and on a suitable adaptation of conservativity to a proof-theoretic setting, it was proved in [3] that

every determiner is conservative (in at least one of its arguments).

Thus, conservativity cannot serve as a criterion for filtering determiners meanings.

However, there are expressions that have the above proof-theoretic type, but still *cannot* be considered as NL-realisable determiners.

In this paper, I want to argue for another filtering property of determiners-like meanings, based on their proof-theoretic characterisation, to serve as a necessary criterion for admissibility as an NL-realised determiner.

[1] All NL examples are displayed in San-serif font and are mentioned, not used.