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Tableau reductions: Towards an optimal decision procedure for the modal necessity ☆

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ABSTRACT

We present a new prefixed tableau system TK for verification of validity in modal logic K. The system TK is deterministic, it uniquely generates exactly one proof tree for each clausal representation of formulas, and, moreover, it uses some syntactic reductions of prefixes. TK is defined in the original methodology of tableau systems, without any external technique such as backtracking, backjumping, etc. Since all the necessary bookkeeping is built into the rules, the system is not only a basis for a validity algorithm, but is itself a decision procedure. We present also a deterministic tableau decision procedure which is an extension of TK and can be used for the global assumptions problem.

1. Introduction

In the last decades, logics – and modal logics in particular – have become very popular and extremely useful in many areas of computer science. These areas include information security, knowledge and belief representation, database theory and distributed systems, program verification, cryptography and agent based systems, computational linguistics, and nonmonotonic formalisms.

There are several approaches for logics in security information systems. For instance, in [7], a logic for specifying and reasoning about secure distributed systems is described; this logic combines modalities for knowledge and time with modalities for permission and obligation. In [1], some of the concepts, protocols, and algorithms for access control in distributed systems from a logical perspective are studied. In [12], the influence of trust on the assimilation of acquired information into an agent's belief is considered by using modal logic, and the relationship between belief, information acquisition, and trust is characterized; moreover, applications to computer security and database reasoning are also suggested. More recently, in [3],

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a formal language with modal logic to prevent unauthorized and malicious access to information systems is proposed, where knowledge related modal operators are employed to represent agents' knowledge in reasoning. Furthermore, in [4], a version of distributed temporal logic is introduced, that is well-suited both for verifying security protocols and as a metalogic for reasoning about different security protocol models. Recent advances in the area of security for information systems can be found, for instance, in [2,5].

The growing importance of applications of modal logics in AI has led to an increased interest in the design of optimal and effective deduction systems for modal logics, and – in the case of decidable logics – of effective and simple-to-use decision procedures (cf. [6]). The minimal normal modal logic is the logic K of the modal necessity. The most popular deduction systems for K are designed in tableau style. Each tableau proof system for K can be seen as the core of all tableau systems for normal modal logics, and hence as the generic reasoning method for many logic based approaches for security in information systems. The famous tableau system for K is Fitting's tableau (see e.g., [10]). It is obtained from the tableau for classical propositional logic by adding two rules: the reduction rule (ϑ) and the necessitation rule (κ). Both these rules are nondeterministic, since their application depends on the choice of the formula to which they are applied. This means that among all the possible decomposition trees of a valid formula some of them are closed, that is they are proofs of the formula, while the others are open, that is they are not proofs of the formula. Therefore, in the general case, to verify validity of a formula, we must check all possible tableau trees, until we find a closed one. However, a tableau system does not include any rule for finding the proper tree. Within the framework of Fitting's tableau calculus it is impossible to avoid this nondeterminism. To solve this problem, the prover needs to backtrack over nondeterministic applications of the rules. Furthermore, the Fitting's tableau has also another kind of nondeterminism. We must care not only to which formula we apply the rules, but also at which state we do so, since the preceding application of the rules might delete information which is necessary for finding a closed tree. Hence, one of the main challenges in designing a generic tableau prover is to identify and to remove the nondeterminism implicit in the pen-and-paper formulation of tableau systems. To obtain a decision procedure, inference rules need to be complemented by a *strategy of finding a proof*. Thus, in the case of modal logics, tableau systems are not their decision procedures, they only provide a basis on which decision procedures can be developed.

The aim of this paper is to present a new tableau system TK for modal logic K. The motivation for considering the system TK is rather theoretical than practical. We are interested in a tableau which is not only a basis for an algorithm verifying validity of a formula, but is *itself* a deterministic decision procedure. To be precise, by a tableau we mean a system determined by rules and axioms. Having this notion of tableau system, any extension of a tableau with additional deduction machinery for proof searching, such as backtracking, backjumping or simplifications, is not a tableau in our sense. Second, by a decision procedure we mean an algorithm which decides whether a formula is valid or not in a finite number of steps. Thus, we are interested in the 'traditional' tableau which is itself an algorithm for deciding validity of formulas. That is, the tableau algorithm has to be of the following type: construct *any* proof tree in the tableau and then you will know the answer whether a formula is valid or not. With these assumptions, any extension of tableaux with external techniques is in our terminology a decision procedure *based* on tableaux. Of course, we are aware that the elimination of additional tools in the construction of a decision procedure may increase the space complexity. It is the cost of limitations on the construction of an algorithm to the pure tableau methodology. That is, if additional tools are allowed, a possible algorithm may be better than that constructed with the use of weaker tools. However, our aim is not to construct any *tableau-based decision procedure*, possibly with the best space complexity. Our aim is to construct the best possible *tableau decision procedure* in the sense explained above. The problem of existence of such a decision procedure with good space complexity is interesting enough from a theoretical point of view. However, it may be also noteworthy for practical reasons, since in practice deterministic decision procedures may be more effective than non-deterministic ones.