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The existence of superluminal particles is consistent with relativistic dynamics

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ABSTRACT

Within an axiomatic framework, we prove that the existence of faster than light (FTL) particles is consistent with (does not contradict) the dynamics of Einstein's special relativity. The proof goes by constructing a model of relativistic dynamics where FTL particles can move with arbitrary speeds. To have a complete picture, we not only construct an appropriate model but explicitly list all the basic assumptions (axioms) we use.

1. Introduction

From time to time certain experiments (such as OPERA 2011, MINOS 2007, etc.) appear suggesting that there may be faster than light (FTL) particles. Almost all of these experiments turned out to be erroneous so far. However, the tendency that these experiments usually turn out to be erroneous gives us no guarantee that there will be no experiment in the future justifying the existence of FTL particles. Also Recami's recent overview [30] contains some experimental sectors of physics still suggesting the existence of FTL objects.

Anyway, if we have a reliable experiment showing the existence of FTL particles, we have to rebuild or modify all the theories inconsistent with (contradicting) FTL motion. Weinberg–Salam theory is a good example of such a theory because it implies the impossibility of FTL motion [25].

In this paper, we show that the particle dynamics of Einstein's special relativity would survive any experiment showing the existence of FTL objects because it is logically consistent with their existence. The only framework for investigating the consistency of a statement with a theory is the axiomatic framework of mathematical logic. Therefore, we investigate the consistency of FTL particles in the framework of mathematical logic.

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The investigation of FTL motion goes back to pre-relativistic times, see, e.g., [16], [28, §3]. Since 1905 it has generally been believed that the nonexistence of FTL particles is a direct consequence of special theory of relativity. Since Tolman's antitelephone argument [37], several paradoxes concerning causality violations and FTL particles have appeared, and since the 1950s great many papers have been published on theories for FTL particles as well as on possible resolutions of the paradoxes, see, e.g., [8,9,11,17–21,26–29,31–35,38,39], and references therein.

Since causality paradoxes are based on changing the past some way, they are usually resolved by making restrictions on the things that can be changed in the corresponding situations, see, e.g., Novikov's self-consistency principle [15,22]. The possible resolution of causal paradoxes has an extensive literature. Moreover, our research group showed in [7], that FTL motion does not imply that information can be sent to the past even if we assume that there are FTL particles moving with arbitrary speeds. Therefore, instead of investigating the FTL motion based causal paradoxes, here we concentrate only to the more basic question whether relativistic dynamics allows the existence of massive FTL particles or not.

To show that relativistic dynamics allows the existence of massive FTL particles, we have to construct a model of relativistic dynamics where there are such particles. However, to have a complete picture, not only the model construction is important but the basic assumptions (axioms) we take. Therefore, we introduce an axiomatic theory of relativistic dynamics (SRDyn) and show that this axiom system has an appropriate model.

As far as we know, apart from ours, none of the theories for FTL particles in the literature is truly axiomatic in the sense of mathematical logic. A key feature of working within a truly axiomatic theory lies in the fact that within such a theory no tacit assumptions are allowed, all the assumptions have to be revealed as formal axioms. This feature is crucial in investigating consistency questions as well as any other foundational questions because in these investigations we have to see clearly what is being assumed and what is not.

In an axiomatic framework similar to the one used here, [36] shows that the existence of FTL inertial particles does not contradict (i.e., it is consistent with) special relativistic kinematics. In other words, there is a model of relativistic kinematics containing FTL particles. This means exactly that the existence of FTL particles is logically independent of relativistic kinematics because, of course, there is also a model of relativistic kinematics in which there are no FTL particles.

In this paper, we show that the existence of massive FTL inertial particles is logically independent of special relativistic dynamics, too. This means that relativistic dynamics implies neither the nonexistence nor the existence of massive FTL particles; or equivalently both the existence and the nonexistence of massive FTL particles are consistent with relativistic dynamics.

This situation is completely analogous to the fact that Euclid's postulate of parallels is logically independent of the rest of its axioms (in this case two different consistent theories extending the theory of absolute geometry are Euclidean geometry and hyperbolic geometry).

Based on Einstein's original 1905 postulates, we formalize the dynamics of special relativity within an axiomatic framework. We chose first-order logic to formulate axioms of special relativity because experience (e.g., in geometry and set theory) shows that this logic is an adequate logic for providing axiomatic foundations for a theory.

To create any theory of FTL particles, we have to deal with the following phenomenon implied already by the kinematics of special relativity. If an observer sees a fusion of two particles in which an FTL particle participates, then a fast enough (but slower than light) observer sees this fusion as a decay, see Fig. 1. The same example also appears, e.g., in [9,28,34] and in connection with the phenomenon [9] says: *"... according to the original criteria, various observers must agree on the identity of physical laws, and not the description of any given phenomenon ..."*. So the existence of FTL particles adds new concepts to the already long list of observer dependent concepts of relativity theory, namely it is also observer dependent whether a particle participates in a decay or a fusion.