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On the a priori and a posteriori assessment of probabilities

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ABSTRACT

We argue that in spite of their apparent dissimilarity, the methodologies employed in the a priori and a posteriori assessment of probabilities can both be justified by appeal to a single principle of inductive reasoning, viz., the principle of symmetry. The difference between these two methodologies consists in the way in which information about the single-trial probabilities in a repeatable chance process is extracted from the constraints imposed by this principle. In the case of a posteriori reasoning, these constraints inform the analysis by fixing an a posteriori determinant of the probabilities, whereas, in the case of a priori reasoning, they imply certain claims which then serve as the basis for subsequent probabilistic deductions. In a given context of inquiry, the particular form which a priori or a posteriori reason may take depends, in large part, on the strength of the underlying symmetry assumed: the stronger the symmetry, the more information can be acquired a priori and the less information about the long-run behavior of the process is needed for an a posteriori assessment of the probabilities. In the context of this framework, frequency-based reasoning emerges as a limiting case of a posteriori reasoning, and reasoning about simple games of chance, as a limiting case of a priori reasoning. Between these two extremes, both a priori and a posteriori reasoning can take a variety of intermediate forms.

1. Introduction

In the context of elementary games of chance, such as those which involve the tossing of coins, the casting of dice, or the drawing of cards from a shuffled deck, the assessment of probabilities can proceed in one of two ways: either by means of an 'a priori' assignment of equal probability to each of the possible outcomes of a given chance process, or else on the basis of an 'a posteriori' estimate of the long-run relative frequency with which a given event will occur.[2] Suppose, for example, that we are casting a die and that we wish to assess how likely it is that on a given toss, at least three points will be scored. To do so, we may either appeal directly to the physical symmetry of the die in order to justify an assignment of equal probability to each of the six possible outcomes of the cast (and hence a probability of 2/3 to the event in question), or else we may toss the die repeatedly and observe how often at least three points are scored in a sufficiently large number of repeated trials.

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[1] An early version of this paper was presented at the 2011 Progic Conference in honor of Haim Gaifman's contributions to the philosophy of probability and logic. It was, for me, a particular pleasure to be able to participate in this conference since Professor Gaifman was the principal supervisor of my graduate research at Columbia University, during which time the ideas discussed in the present work first began to take shape.

[2] The use of the terms 'a priori' and 'a posteriori' as a means of distinguishing between the judgments that result from these two methodologies has a long history dating back to the earliest work in probability theory. It is in keeping with this precedent that I have opted to employ this terminology. Nevertheless, so as to avoid any confusion, it should be stressed right away that these terms are not to be understood in their traditional philosophical sense. In particular, the term 'a priori' does not imply that the assumptions which underwrite a priori assessments of probability are devoid of empirical content. It is rather meant to express the fact that these judgments are based on knowledge acquired by the agent independently of (or 'prior to') any direct observations made of the particular chance process under study.

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On first inspection, these two methodologies appear to be based on inductive principles of an altogether different sort. Thus, for example, whereas the notion of symmetry figures explicitly in the a priori assessment of probabilities, it appears to play no role at all in a posteriori reasoning, and the same appears to be true, only in reverse, with respect to the notion of a repeated trial. What then are we to make of this apparent methodological duality? Are there, in fact, two fundamentally different sorts of knowledge from which probabilities can be inferred?[3] In this paper, we argue that the answer to this question is no. When properly understood, the difference between the assumptions underwriting a priori and a posteriori assessments of probability turns out to be a difference of degree and not of kind. In defense of this claim, I will outline a general model of probabilistic inference within which both a priori and a posteriori methods can be viewed as particular instances of a single overarching pattern of symmetry-based reasoning.

The mathematical facts which will allow us to effect such a reduction are not new; they are elementary results in what is termed ergodic theory, the theory which investigates the asymptotic behavior of measure preserving transformation groups. While ergodic theory is, at present, a well-developed branch of mathematics, and its significance for a general theory of probabilistic reasoning has long been acknowledged and thoroughly explored,[4] there remain parts of the story that have not yet been told. In particular, the interpretation offered below of a priori probabilities as those which can be judged true of any single trial in an infinitely repeatable chance process solely in virtue of the probabilistic symmetries exhibited by that process, has, to the best of my knowledge, not been considered. And yet, as we shall see, such an interpretation gives rise to a number of questions that are likely to be of interest to both philosophers and technicians alike.

The structure of the paper is as follows. In Section 2, we provide a brief account of the general logic of symmetry-based reasoning and explain how such reasoning can be applied in the context of an infinitely repeatable experiment modeled as a discrete-time stochastic process. In Section 3, we make precise the sense in which both a priori and a posteriori methods can be interpreted as particular instances of such symmetry-based reasoning. As is discussed in Section 4, such an analysis not only makes clear the precise sense in which both frequency-based reasoning and reasoning about simple games of chance represent degenerate forms of probabilistic reasoning, but also leads naturally to the identification of 'intermediate' contexts of inquiry in which the assessment of probabilities proceeds in part on the basis of a priori, and in part on the basis of a posteriori reasoning. In Section 5, we offer some concluding remarks.

2. The logic of symmetry-based reasoning

Suppose that a die is about to be cast on a table and that we wish to assign probabilities to each of the six possible outcomes of the cast. The most obvious way to do so is by appeal to the physical symmetry of the die. An ordinary six-sided die is cubic in shape, and, as such, were it to be rotated in either direction by any multiple of 90° about an axis of rotation passing through the die's center and orthogonal to any one of its faces, the mass distribution of the die would remain unchanged. Provided, therefore, it can be assumed that all that is relevant for assessing the probability with which the die will land in a given position is contained in a description of its mass distribution once it has come to rest, we can justify assigning an equal probability to each of the six possible outcomes of the cast on the grounds that any one such outcome can be obtained from any other via one or more rotations of the above sort.

This is a probabilistic argument from symmetry. Such arguments, in general, proceed from an initial assumption to the effect that a certain of way of transforming a given chance process leaves unaltered all that is relevant for assessing the probabilities of its possible outcomes. On the basis of this assumption, it is then concluded that if one such outcome can be obtained from another by means of such a structure-preserving transformation, then these two outcomes must have the same probability.

In formalizing such arguments, the class of structure-preserving transformations is represented by a symmetry group, i.e., a class of bijective mappings ($\mathscr{T}$) from the set of possible outcomes of a chance process (Ω) to itself, which forms a mathematical group under the operation of function composition. The functions in $\mathscr{T}$ are interpreted as ways of transforming the process that leave unaltered all the probabilistically relevant aspects of the situation. Thus, two outcomes, $\omega, \omega' \in \Omega$ agree in all relevant respects if

$$\omega' = T(\omega), \tag{2.1}$$

[3] In response to this apparent methodological dualism, the most common approach among contemporary philosophers of probability has been to accept a posteriori reasoning as the only valid form of probabilistic induction, and to then explain the success of a priori reasoning in the context of simple games of chance by reducing it to a form of deduction from prior probabilistic assumptions themselves justified on the basis of an a posteriori inference from previous observations. See, e.g., Strevens [14].

It is perhaps interesting to note, however, that among the earliest theorists of probability, the opposite approach was adopted. While it was generally acknowledged among these thinkers that a priori methods possess a severely limited scope of applicability, there was never any question as to their legitimacy in the context of elementary games of chance. On the contrary, it was a posteriori reasoning, as applied in these settings, that was regarded as the method of second resort, and that called for further justification. (This contradicts the claim made in Carnap [2] that the term a priori probability was traditionally used "… in cases where the evidence …was very weak or even tautological (a statement of 'ignorance')…" (p. 188).) The earliest attempt to provide such a justification appears in the work of Jacob Bernoulli, who proposed to ground such reasoning on what we now recognize to be the first limit theorem in probability theory, an integral form of the weak law of large numbers. For a discussion of Bernoulli's theorem and its historical context, see Hald [9], Ch. 16.

[4] See, e.g., von Plato [16,17], and Dawid [3].