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RIESZ IDEMPOTENT OF (n,k)-QUASI-$*$-PARANORMAL OPERATORS*

Qingping ZENG (曾清平)
College of Computer and Information Sciences, Fujian Agriculture and Forestry University, Fuzhou 350002, China
E-mail: *zqpping2003@163.com*

Huaijie ZHONG (钟怀杰)
School of Mathematics and Computer Science, Fujian Normal University, Fuzhou 350007, China
E-mail: *zhonghuaijie@sina.com*

Abstract A bounded linear operator T on a complex Hilbert space H is called (n,k)-quasi-$*$-paranormal if

$$\|T^{1+n}(T^kx)\|^{1/(1+n)}\|T^kx\|^{n/(1+n)} \geq \|T^*(T^kx)\| \quad \text{for all} \quad x \in H,$$

where n,k are nonnegative integers. This class of operators has many interesting properties and contains the classes of n-$*$-paranormal operators and quasi-$*$-paranormal operators. The aim of this note is to show that every Riesz idempotent E_λ with respect to a non-zero isolated spectral point λ of an (n,k)-quasi-$*$-paranormal operator T is self-adjoint and satisfies $\operatorname{ran}E_\lambda = \ker(T-\lambda) = \ker(T-\lambda)^*$.

Key words $*$-class A operator; $*$-paranormal operator; Riesz idempotent

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1 Introduction

There is a growing interest concerning nonhyponormal operators. Let $L(H)$ be the C^* algebra of all bounded linear operators on an infinite dimensional complex Hilbert space H. Below we list some of these nonhyponormal operators. Recall that an operator $T \in L(H)$ is said to be

- $*$-class A if $|T^2| \geq |T^*|^2$ (see [5]);
- quasi-$*$-class A if $T^*|T^2|T \geq T^*|T^*|^2T$ (see [11]);
- k-quasi-$*$-class A if $T^{*k}|T^2|T^k \geq T^{*k}|T^*|^2T^k$ (see [9]);
- $*$-paranormal if $\|T^2x\|^{1/2}\|x\|^{1/2} \geq \|T^*x\|$ for all $x \in H$ (see [2]);
- quasi-$*$-paranormal if $\|T^2(Tx)\|^{1/2}\|Tx\|^{1/2} \geq \|T^*(Tx)\|$ for all $x \in H$ (see [10]);
- n-$*$-paranormal if $\|T^{1+n}x\|^{1/(1+n)}\|x\|^{n/(1+n)} \geq \|T^*x\|$ for all $x \in H$ (see [8]),

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here and henceforth, n, k denote nonnegative integers.

As an extension of the above operator classes, we introduced and studied in [21] the following definition.

Definition 1.1 An operator $T \in L(H)$ is said to be (n, k)-quasi-$*$-paranormal if

$$\|T^{1+n}(T^k x)\|^{1/(1+n)}\|T^k x\|^{n/(1+n)} \geq \|T^*(T^k x)\| \text{ for all } x \in H.$$

The class of (n, k)-quasi-$*$-paranormal operators has many interesting properties (see [21]), such as inclusion relations, SVEP (single valued extension property), matrix representation, joint point spectrum, and so on.

In the present note, we continue to investigate the properties of (n, k)-quasi-$*$-paranormal operators. We show that every Riesz idempotent E_λ with respect to a non-zero isolated spectral point λ of an (n, k)-quasi-$*$-paranormal operator T is self-adjoint and satisfies $\text{ran}E_\lambda = \ker(T - \lambda) = \ker(T - \lambda)^*$.

2 Riesz Idempotent of (n, k)-Quasi-$*$-Paranormal Operators

The self-adjointness of Riesz idempotent with respect to the isolated spectral point of an operator was investigated by a number of mathematicians around the world. For an isolated spectral point $\lambda \in \text{iso}\sigma(T)$, the Riesz idempotent E_λ with respect to λ is defined by

$$E_\lambda := \frac{1}{2\pi \mathrm{i}} \int_{\partial D} (z - T)^{-1} \mathrm{d}z,$$

where D is a closed disk with center λ and its radius is small enough such that $D \cap \sigma(T) = \{\lambda\}$. In general, the Riesz idempotent E_λ is not orthogonal and E_λ is orthogonal if and only if E_λ is self-adjoint. Stampfli [12] showed that the Riesz idempotent E_λ for an isolated spectral point λ of a hyponormal operator T is self-adjoint. Stampfli's result was extended to p-hyponormal operators and log-hyponormal operators by Chō and Tanahashi [4], to M-hyponormal operators by Chō and Han [3], to $*$-paranormal operators by Tanahashi and Uchiyama [14]. In the case $\lambda \neq 0$, Stampfli's result was extended to p-quasihyponormal operators by Tanahashi and Uchiyama [15], to (p, k)-quasihyponormal operators by Tanahashi, Uchiyama and Chō [16], to w-hyponormal operators by Han, Lee and Wang [6], to class A operators by Uchiyama and Tanahashi [18], to quasi-class A operators by Jeon and Kim [7], to quasi-class (A, k) operators by Tanahashi, Jeon, Kim and Uchiyama [13], to paranormal operators by Uchiyama [17], to k-quasi-$*$-class A operators by Mecheri [9].

In this section, we will extend Stampfli's result to n-$*$-paranormal operators and, to (n, k)-quasi-$*$-paranormal operators in the case $\lambda \neq 0$.

Theorem 2.1 (1) Let T be (n, k)-quasi-$*$-paranormal and $0 \neq \lambda \in \text{iso}\sigma(T)$. Then the Riesz idempotent E_λ is self-adjoint and

$$\text{ran}E_\lambda = \ker(T - \lambda) = \ker(T - \lambda)^*.$$

(2) Let T be n-$*$-paranormal and $\lambda \in \text{iso}\sigma(T)$. Then the Riesz idempotent E_λ is self-adjoint and

$$\text{ran}E_\lambda = \ker(T - \lambda) = \ker(T - \lambda)^*.$$